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Moment Approximation in Product Form Queueing Networks

M. Mittler *and* **R. Müller**

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Bayerische Julius-Maximilians-Universität Würzburg,
Lehrstuhl für Informatik III, Am Hubland, D – 97074 Würzburg,
Tel.: +49-931-8885518, Fax: +49-931-8884601,
e-mail: mittler@informatik.uni-wuerzburg.de

Abstract: *In closed product form queueing networks, the classical Mean Value Analysis (MVA) is an useful tool for the calculation of mean sojourn times and mean number of customers in each queue. MVA has been extended in two directions. Firstly, due to tremendous memory requirements of MVA, approximation methods such as the well-known Self Correcting Approximation Technique (SCAT) have been developed. Secondly, the Moment Analysis (MA) is a generalization of MVA which also allows the computation of higher moments of the population at a particular node of the network.*
To complete the picture, we propose an approximation method for higher moments which will be referred to as SCAT-MA (Self Correcting Approximation Technique – Moment Analysis). SCAT-MA is obtained by taking the derivative of the SCAT equations with respect to the service rates. By means of a more sophisticated example for a closed product form queueing network, we evaluate the performance of our method by comparing exact results to approximate results.

Keywords: *queueing analysis, moment approximations, closed product form queueing networks*

1. Introduction

A large number of publications on solution methods for product form queueing networks has emerged during the last two decades (see e.g. de Souza e Silva and Muntz [7] or Bolch [3]). One way to classify these methods is to divide them into exact and approximate methods. On the other hand one may distinguish between methods that allow the computation of mean values only and methods allowing the calculation of higher moments or distribution functions as well. Certainly, there are further features to classify solution algorithms, among those are for example the computational complexity and the explicit computation of the normalization constant. In this paper, we only consider algorithms which avoid the computation of the normalization constant.

The most popular exact solution method for closed product form queuing networks is the classical Mean Value Analysis (MVA, Reiser and Lavenberg [10]). By means of MVA the first moment of sojourn times and of the number of customers in each queue can be obtained. There are many extensions of MVA considering the type of service centers that have product form or multiple customer classes and so on (see for example de Souza e Silva and Muntz [7] and Bolch [3]). The Moment Analysis (MA, Strelen [14]) is a generalization of MVA that also allows the derivation of higher moments of the population at any queueing station. An alternative method to Strelen's algorithm was published by de Souza e Silva and Muntz [6]. Both methods obtain recursive formulae for the moments of queue lengths. The difference among the two methods is that Strelen takes the derivative of the main MVA equations with respect to the service rates of the service centers whereas de Souza e Silva and Muntz take the derivative of the normalization constant with respect to the visit ratio of a chain k customer to a particular service center. Another method to gain higher moments of queue lengths is the Distribution Analysis by Chain (DAC, de Souza e Silva and Lavenberg [5]). In this case, joint queue length distributions can be computed by means of a recursive procedure which avoids the computation of the normalization constant.

Due to high memory requirements of exact solution algorithms, approximation methods have been developed in order to compute mean values. Among them is the well known Self Correcting Approximation Technique (SCAT, Neuse and Chandy [9]) which is simply an approximation of MVA.

We extend SCAT to the derivation of higher moments of the population sizes at particular nodes and call the new method SCAT-MA (Self Correcting Approximation Technique – Moment Analysis). This goal is achieved by taking the derivatives of the SCAT equations with respect to the service rates. SCAT-MA applies to a class of multiple closed chain product form queueing networks with single servers and fixed service rates. The service centers covered by this class of networks include the queueing nodes (1) $M/M/1$-FCFS, (2) $M/G/1$-PS, (3) $M/G/\infty$, and (4) $M/G/1$-LCFSPR. Furthermore, our method also applies to service centers for which the mean response time and the derivative of the response time with respect to the service rate are available. Without a loss of generality, we assume that customers do not change their class during their visit to the network. For reasons of convenience, we then can use the terms "class of customers" and "chain of customers" equivalently.

The following notation is used throughout the paper:

N : number of service centers,
$\mathcal{N}$: set of service centers, $\mathcal{N} = \{1, \ldots, N\}$,
R : number of customer classes,
$\mathcal{R}$: set of customer classes, $\mathcal{R} = \{1, \ldots, R\}$,
K_{ir} : population of chain r at service center i,
K_r : population of chain r, $K_r = \sum_{i \in \mathcal{N}} K_{ir}$,
K : number of customers in the network, $K = \sum_{r \in \mathcal{R}} K_r$,
$\underline{K}$: population of the network, $\underline{K} = (K_1, \ldots, K_R)$,
λ_r : mean throughput of chain r,
μ_{ir} : service rate of class r customers at node i,
θ_{ir} : visit ratio of chain r to node i,
L_{ir} : random variable for the number of class r customers at node i,
L_i : random variable for the number of customers at node i, $L_i = \sum_{r \in \mathcal{R}} L_{ir}$,
T_{ir} : random variable for the response time (= waiting time + service time) of class r customers at node i,
$\mathsf{E}[X]$: mean of the random variable X,
$\overline{X}$: alternative notation to $\mathsf{E}[X]$,
$\mathsf{Var}[X]$: variance of the random variable X,
$\mathsf{Cov}[X, Y]$: covariance of the random variables X and Y,
δ_{ir} : $\delta_{ir} = 1$ if $i = r$ and $\delta_{ir} = 0$ else,
$\underline{e}_r$: rth unit column vector, $\underline{e}_r = (\delta_{1r}, \ldots, \delta_{Rr})^T$.

The paper is organized as follows. First, we revisit the original algorithms of MVA, MA, and SCAT in Sec. 2. We extend SCAT to the analysis of higher moments and obtain SCAT-MA in Sec. 3. The accuracy of our new algorithm is stressed in Sec. 4, where approximation results achieved by using SCAT-MA are compared to exact results obtained by means of MA.

2. Generalization and Approximation of MVA

The original MVA is due to Reiser and Lavenberg [10] and applies to mixed product-form queueing networks if the network contains at least a single closed chain of customers. The MVA is based on the arrival theorem (see Lavenberg and Reiser [8] and Sevcik and Mitrani [12]), which states that in a closed queueing network the stationary state probabilities at arrival instants q_n correspond to the stationary state probabilities p_n with one customer removed from the network, i.e. $q_n(K) = p_n(K-1)$, $0 \leq n \leq K-1$.

Given any population $\underline{k}$, it can be shown that

$$\overline{T}_{ir}(\underline{k}) \quad = \quad \frac{1}{\mu_i} \cdot \left[1 + \sum_{s=1}^{R} \overline{L}_{is}(\underline{k} - \underline{e}_r) \right]. \tag{2.1}$$

This equation is self-explaining for single server FCFS nodes since the mean response time of a tagged customer at a node is simply the mean service time for all customers present at the arrival instant of the tagged customer plus the mean service time for the tagged customer. Certainly, the mean response time of eqn. (2.1) has to be modified for each of the BCMP queueing nodes (cf. [10]).

By Little's law the throughput of each customer class is

$$\lambda_r(\underline{k}) \quad = \quad \frac{K_r}{\sum\limits_{i=1}^{N} \theta_{ir} \cdot \overline{T}_{ir}(\underline{k})}. \tag{2.2}$$

From this equations follows (once again by Little's law) that the mean number of class r customers at node i meets the following equation:

$$\overline{L}_{ir}(\underline{k}) \quad = \quad \lambda_r(\underline{k}) \cdot \overline{T}_{ir}(\underline{k}) \cdot \theta_{ir}. \tag{2.3}$$

The recursive solution of eqns. (2.1), (2.2), and (2.3) beginning with population $\underline{k} = (0, \ldots, 0)$ yields an exact analysis of the system when the maximum population $\underline{k} = \underline{K}$ is reached.

MVA yields only the mean response time and the mean number of customers present at any of the service centers. By taking the ν-th derivative of the MVA equations (2.1 – 2.3) with respect to the service rates, Strelen [14] presented a recursive algorithm analogous to MVA so as to derive the ν-th moment of the population at the nodes of the network. Strelen showed that the variance and the covariance of the number of customers meet the following equations:

$$\mathsf{Var}[L_{ir}] \quad = \quad \mathsf{E}[L_{ir}^2] - (\mathsf{E}[L_{ir}])^2 \quad = \quad -\mu_{ir} \cdot \frac{\partial}{\partial \mu_{ir}} \overline{L}_{ir}(\underline{K}), \tag{2.4}$$

$$\mathsf{Cov}[L_{ir}, L_{js}] \quad = \quad \mathsf{E}[L_{ir} \cdot L_{js}] - \mathsf{E}[L_{ir}] \cdot \mathsf{E}[L_{js}] \quad = \quad -\mu_{ir} \cdot \frac{\partial}{\partial \mu_{ir}} \overline{L}_{js}(\underline{K}). \tag{2.5}$$

The unknown derivatives of the main MVA equations on the right hand side of eqns. (2.4) and (2.5) are

$$\frac{\partial}{\partial \mu_{js}} \overline{T}_{ir}(\underline{k}) \quad = \quad \begin{cases} 0 & K_r = 0, \\ -\delta_{ij} \cdot \dfrac{\theta_{ir}}{\mu_{ir}^2} & \text{node type (3)}, \\ \dfrac{\theta_{ir}}{\mu_{ir}^2} \cdot \left[-\delta_{ij} \cdot \Big(1 + \overline{L}_i(\underline{k} - \underline{e}_r) \Big) + \mu_{ir} \cdot \dfrac{\partial}{\partial \mu_{js}} \overline{L}_i(\underline{k} - \underline{e}_r) \right] & \\ & \text{node types (1),(2), (4)}, \end{cases} \tag{2.6}$$

$$\frac{\partial}{\partial \mu_{js}} \lambda_r(\underline{k}) = \frac{-K_r \sum\limits_{i \in S(r)} \frac{\partial}{\partial \mu_{js}} \overline{T}_{ir}(\underline{k})}{\left[\sum\limits_{i \in S(r)} \overline{T}_{ir}(\underline{k}) \right]^2}, \tag{2.7}$$

$$\frac{\partial}{\partial \mu_{js}} \overline{L}_{ir}(\underline{k}) = \theta_{ir} \cdot \left\{ \frac{\partial}{\partial \mu_{js}} \lambda_r(\underline{k}) \cdot \overline{T}_{ir}(\underline{k}) + \lambda_r(\underline{k}) \cdot \frac{\partial}{\partial \mu_{js}} \overline{T}_{ir}(\underline{k}) \right\}. \tag{2.8}$$

where $S(r)$ is the set of nodes visited by class r customers.

Strelen [14] obtained an iterative procedure analogous to the original MVA for the computation of the differentiated response times, the differentiated throughputs, and the differentiated mean population sizes. By this means the variables of the left hand side of eqns. (2.4) and (2.5) can easily be calculated.

Since the memory requirements of MVA grow combinatorically with the number of customers and with the number of nodes, the application of MVA to complex problems is very limited. For this reason approximation methods have been developed. Following Bard [2] and Schweitzer [11], in single server closed queueing networks the basic idea behind the approximation methods is to approximate the mean population of the network given population $\underline{K}$ with one customer removed from the network. This yields an iterative procedure which stops when a certain approximation accuracy is achieved. Bard and Schweitzer used the following approximation of the mean population:

$$\overline{L}_{ir}(\underline{K} - \underline{e}_s) = \frac{(\underline{K} - \underline{e}_s) \cdot \underline{e}_r}{K_r} \cdot \overline{L}_{ir}(\underline{K}). \tag{2.9}$$

It is obvious that for large populations the right hand side of eqn. (2.9) does not change significantly if one customer is removed from the network.

Neuse and Chandy [9] extended this method to the Self Correcting Approximation Technique (SCAT) considering multi-server queueing networks. In addition to the approximation of Bard and Schweitzer they also consider the difference of the mean population size from one iteration step to the next. We briefly review SCAT and define

$$F_{ir}(\underline{K}) = \frac{\overline{L}_{ir}(\underline{K})}{K_r} \tag{2.10}$$

as the portion of class r customers given population $\underline{K}$. Then, we get the following difference if we remove one customer of class s from the network:

$$D_{irs}(\underline{K}) = F_{ir}(\underline{K} - \underline{e}_s) - F_{ir}(\underline{K}) = \frac{\overline{L}_{ir}(\underline{K} - \underline{e}_s)}{(\underline{K} - \underline{e}_s) \cdot \underline{e}_r} - \frac{\overline{L}_{ir}(\underline{K})}{K_r}. \tag{2.11}$$

From this two equations an approximation of the mean population size is obtained for a network with one customer of class s removed from population $\underline{K}$:

$$\overline{L}_{ir}(\underline{K} - \underline{e}_s) = (\underline{K} - \underline{e}_s) \cdot \underline{e}_r \cdot \left[F_{ir}(\underline{K}) + D_{irs}(\underline{K}) \right]. \tag{2.12}$$

It is obvious that this method includes the method proposed by Bard and Schweitzer as a special case ($D_{irs}(\underline{K}) = 0$ for $i \in \mathcal{N}$ and $r, s \in \mathcal{R}$). The entire algorithm of SCAT is given in Alg. 2. The core of SCAT is the so-called CORE algorithm (see Alg. 1). SCAT uses the approximations for all populations $(\underline{K} - \underline{e}_s)$ in order to improve the approximation for population $\underline{K}$ as given in Alg. 2. For further details, the interested reader is referred to the original literature.

3. Approximation of Higher Moments: SCAT-MA

Strelen [14] also proposed an approximation method for higher moments of the queue lengths based on the Bard and Schweitzer method. We will refer to this approximation as BS-MA. According to eqn. (2.8) of the moment analysis, removing one customer of class s and taking the derivative of the number of customers with respect to the service rate yields

$$\frac{\partial}{\partial \mu_{jq}} L_i(\underline{K} - \underline{e}_s) \approx \frac{K_s - 1}{K_s} \cdot \frac{\partial}{\partial \mu_{jq}} L_i(\underline{K}) + \sum_{\substack{r=1 \\ r \neq s}}^{R} \frac{\partial}{\partial \mu_{jq}} L_{ir}(\underline{K}), \tag{3.1}$$

$$\text{with} \quad \frac{\partial}{\partial \mu_{jq}} L_i(\underline{K}) = \sum_{r=1}^{R} \frac{\partial}{\partial \mu_{jq}} L_{ir}(\underline{K}), \qquad q \in \mathcal{R}. \tag{3.2}$$

Using these two equations Strelen [14] proposed a procedure not reported here to approximate higher moments of the populations of the nodes. Unfortunately, he has not reported numerical results and has not discussed the accuracy of the method.

Based on this idea we propose an extension of the SCAT algorithm for the approximation of higher moments of the population. We call this method SCAT-MA in order to indicate that this method is a combination of both SCAT and MA. Analogous to the derivation of MA, we start by taking the derivative of eqns. (2.10), (2.11), and (2.12) and obtain the following equations:

$$\frac{\partial}{\partial \mu_{jq}} F_{ir}(\underline{K}) = \frac{\frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}(\underline{K})}{K_r}, \tag{3.3}$$

$$\begin{aligned} \frac{\partial}{\partial \mu_{jq}} D_{irs}(\underline{K}) &= \frac{\partial}{\partial \mu_{jq}} F_{ir}(\underline{K} - \underline{e}_s) - \frac{\partial}{\partial \mu_{jq}} F_{ir}(\underline{K}) \\ &= \frac{\frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}(\underline{K} - \underline{e}_s)}{(\underline{K} - \underline{e}_s) \cdot \underline{e}_r} - \frac{\frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}(\underline{K})}{K_r}, \end{aligned} \tag{3.4}$$

$$\frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}(\underline{K} - \underline{e}_s) = (\underline{K} - \underline{e}_s) \cdot \underline{e}_r \cdot \left[\frac{\partial}{\partial \mu_{jq}} F_{ir}(\underline{K}) + \frac{\partial}{\partial \mu_{jq}} D_{irs}(\underline{K}) \right]. \tag{3.5}$$

Taking into account this result, we modify the algorithms CORE and SCAT and obtain the algorithms CORE-MA and SCAT-MA as shown in Alg. 3 and Alg. 4.

4. Approximation Accuracy: A Case Study

We examine the accuracy of our approximation method by means of a closed multiple chain product form queueing network which is a queueing network representation of the well-known machine interference problem (MIP, see for example [13, 4, 1]). The reason for this choice is that by means of this network we are able to evaluate higher moments of the population for individual classes. Then, we can stress the accuracy of our method for the moments of the total number of customers at a particular node which consists of the moments of individual classes.

4.1 The Machine Interference Problem

The MIP deals with a system consisting of N machines and one or several repairmen where machines are subject to failures. After a machine has broken down it waits for an idle repairman. Then it is serviced and starts producing until the next breakdown occurs. Several queueing disciplines can be used at the service facility. In our study we employed FCFS as queueing discipline at the repair station and have assumed exponentially distributed repair and breakdown times. Further, we have assumed that failure times are identical for all machines.
This problem can be modelled by a closed product form queueing network with N different chains of customers where each chain contains only a single customer. The network consists of $N+1$ nodes where node 0 corresponds to the repair facility and the service times of nodes 1 to N incorporate the failure times of the machines.

The performance measure we examine is the *machine availability* M which denotes the fraction of working machines, i.e.

$$M = 1 - \frac{L_0}{N}. \tag{4.1}$$

Here, L_0 denotes the number of failed machines waiting for repair or receiving repair at the repair station. Please notice that M is a compound random variable consisting of the number of customers of each class present at the repair station. We stress the accuracy of the approximations for the mean and the variance of the machine availability, resp.

4.2 Numerical Results

We examine a symmetric system with eight identical machines and one repairman. In the following graphs, exact and approximate results will be plotted as a function of the fixed repair rate μ of the repair facility. For each performance measure we also plot the relative errors. If X denotes the exact value and Z the approximated, the relative error is defined as the fraction $\frac{Z-X}{X}$.

The approximated means of the machine availability are in very good accordance with the exact results, as can be seen in Figs. 1 and 2. The relative error stays below 1%. A small improvement can be achieved by using a smaller constant ε in the SCAT algorithm. In this example ε was set to the original value given by Chandy and Neuse [9]. Clearly SCAT achieves better results than the Bard and Schweitzer method (in the plots indicated by BS).

In contrast to the mean, significant errors can be observed for the variance of the number of chain 1 customers present at the repair station $\mathsf{Var}[L_{01}]$ (see Fig. 5 and 6). The approximation results for the variances are indicated by the suffix MA. The SCAT-MA approach produces good approximations of the variance of the number of jobs of a certain jobtype (see Fig. 3 and 4). The relative errors are within a few percent and are therefore quite satisfactory for practical purposes. Once again, the approximation based on SCAT performs better than the one based on the Bard and Schweitzer method. Unfortunately, if the approximated second moments are added to compute the variance of the total machine availability, the relative error reaches nearly 20% (see Fig. 6) and further, the BS-MA performs better than SCAT-MA.

In the case of compound random variables, the approximation may be seen as a first shot approach if one is interested in the magnitude of the variance of the machine availability or if the memory requirements exceed the maximum value. Certainly, in some cases an accuracy of 20% may be acceptable. However, based on the moments of population at each node other performance measures (for example moments of sojourn times) have to be evaluated carefully.

5. Conclusion

MVA and its modifications SCAT and MA are standard techniques for the analysis of closed product form queueing networks. Based on SCAT and MA, we presented an approximation method for higher moments called SCAT-MA. The results achieved indicate that our method works very well if only the second moment of the number of customers of a distinct class at a particular node is considered. The approximation accuracy gets worse for the second moment of the total number of customers at a particular node. Nevertheless, SCAT-MA may be seen as a first shot approach if the memory requirements exceed a given capacity. Therefore, the method can adequately be implemented in performance analysis tools for product form queueing networks.

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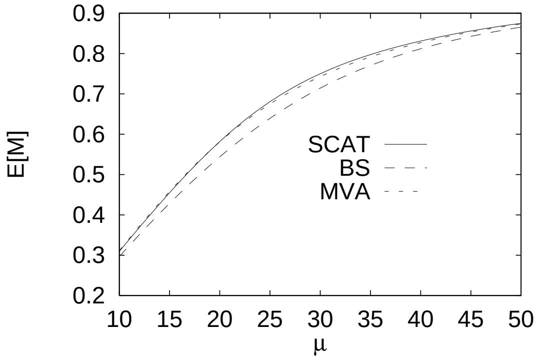

Fig. 1: *Mean machine availability*

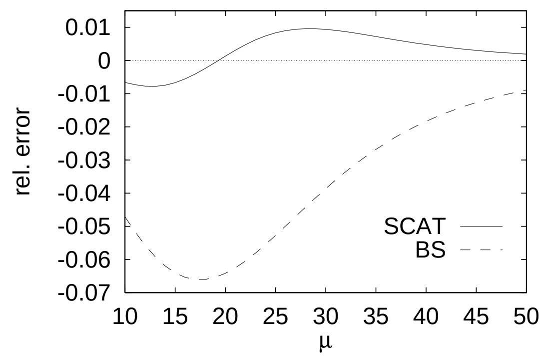

Fig. 2: *Relative error of the mean machine availability*

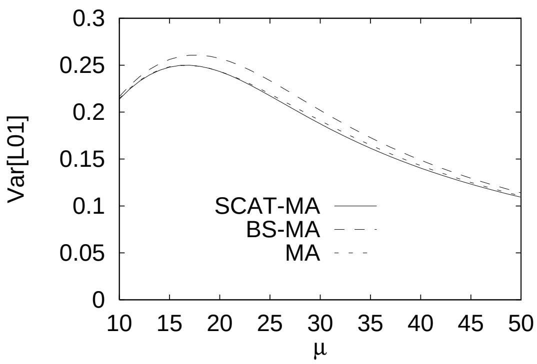

Fig. 3: *Variance of* L_{01}

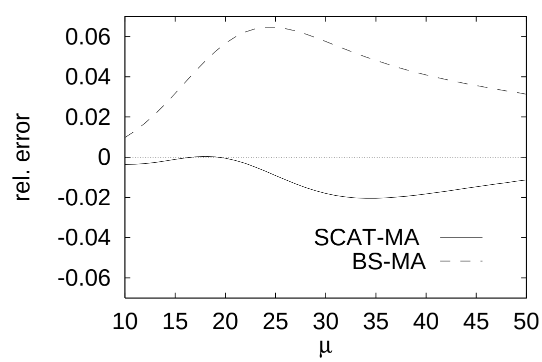

Fig. 4: *Relative error of the variance of* L_{01}

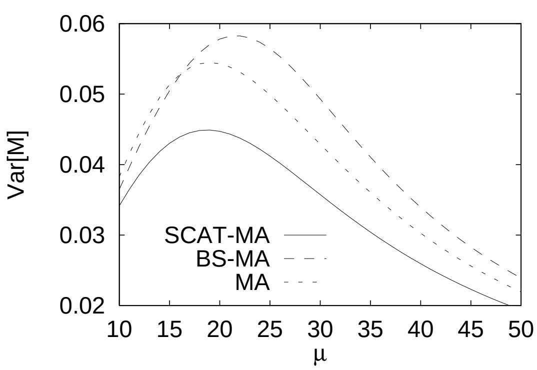

Fig. 5: *Variance of the machine availability*

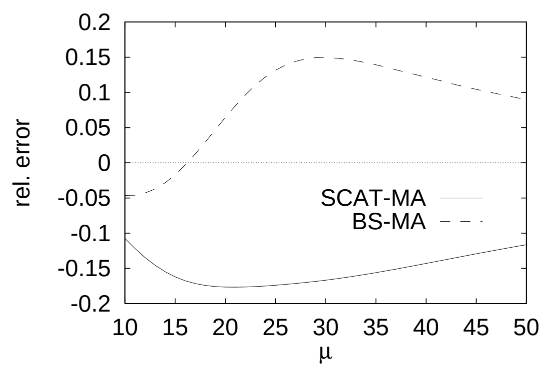

Fig. 6: *Relative error of the variance of the machine availability*

Algorithm 1: Original Version of CORE

(1) Computation of the mean queue lengths with one customer removed from the network according to eqn. (2.12).

(2) Step 2 of MVA with population $\underline{K}$.

(3) Stop if

$$\frac{\left|\overline{L}_{ir}^{(n)}(\underline{K}) - \overline{L}_{ir}^{(n-1)}(\underline{K})\right|}{K_r} \; < \; \varepsilon$$

for all $i \in \mathcal{N}$ and $r \in \mathcal{R}$, else go to step 1.
Here, $\overline{L}_{ir}^{(n)}(\underline{K})$ denotes the mean number of customers in node i of class r at population $\underline{K}$ after the n-th step of iteration.

Algorithm 2: Original Version of SCAT

(1) CORE at population $\underline{K}$ with

$$\overline{L}_{ir}(\underline{K}) \;=\; \frac{K_r}{N} \text{ and } D_{irs}(\underline{K}) \;=\; 0, \quad i \in \mathcal{N}, \;\; r, s \in \mathcal{R}.$$

(2) CORE for $s \in \mathcal{R}$ at population $(\underline{K} - \underline{e}_s)$ with

$$\overline{L}_{ir}(\underline{K} - \underline{e}_s) \;=\; \frac{(\underline{K} - \underline{e}_s) \cdot \underline{e}_r}{N} \text{ and } D_{irj}(\underline{K} - \underline{e}_s) \;=\; 0, \quad i \in \mathcal{N}, \;\; j, r, s \in \mathcal{R}.$$

(3) For all $i, s \in \mathcal{N}$ and $r \in \mathcal{R}$:

  (i) Compute $F_{ir}(\underline{K})$ and $F_{ir}(\underline{K} - \underline{e}_s)$ according to eqn. (2.10).

  (ii) Compute $D_{irs}(\underline{K})$ according to eqn. (2.11).

(4) CORE at population $\underline{K}$ with $\overline{L}_{ir}(\underline{K})$ from step 1 and $D_{irs}(\underline{K})$ from step 3.

Algorithm 3: CORE extended to Moment Analysis (CORE-MA)

(1) Compute $\frac{\partial}{\partial \mu_{jq}} F_{ir}(\underline{K})$ according to eqn. (3.5).

(2) Approximate analysis according to one iteration step of Extended MVA at population $\underline{K}$.

(3) Stop if

$$\frac{\left| \frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}^{(n)}(\underline{K}) \; - \; \frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}^{(n-1)}(\underline{K}) \right|}{K_r} \;<\; \varepsilon$$

for all $i, j \in \mathcal{N}$ and $q, r \in \mathcal{R}$, else go to step 1.

(4) Compute $\frac{\partial}{\partial \mu_{jq}} F_{ir}(\underline{K})$ and $\frac{\partial}{\partial \mu_{jq}} F_{ir}(\underline{K} - \underline{e}_d)$ according to eqn. (3.3), $d \in \mathcal{R}$.

Compute $\frac{\partial}{\partial \mu_{jq}} D_{irs}(\underline{K})$ according to eqn. (3.4).

(5) ExtCORE at population $\underline{K}$.

Algorithm 4: SCAT extended to Moment Analysis (SCAT-MA)

(1) Approximation by means of SCAT.

(2) CORE-MA at population $\underline{K}$ with

$$\frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}(\underline{K}) \;=\; 0 \;\text{ and }\; \frac{\partial}{\partial \mu_{jq}} D_{irs}(\underline{K}) \;=\; 0, \quad i, j \in \mathcal{N}, \;\; r, s \in \mathcal{R}.$$

(3) CORE-MA for $d \in \mathcal{R}$ at population $(\underline{K} - \underline{e}_d)$ with

$$\frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}(\underline{K} - \underline{e}_d) \;=\; 0 \;\text{ and }\; \frac{\partial}{\partial \mu_{jq}} D_{irs}(\underline{K} - \underline{e}_d) \;=\; 0, \quad i, j \in \mathcal{N}, \;\; r, s \in \mathcal{R}.$$

(4) Stop if

$$\frac{\left| \frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}^{(n)}(\underline{K}) \; - \; \frac{\partial}{\partial \mu_{jq}} \overline{L}_{ir}^{(n-1)}(\underline{K}) \right|}{K_r} \;<\; \varepsilon$$

for all $i, j \in \mathcal{N}$ and $q, r \in \mathcal{R}$, else go to step 1. Here, $L_{ir}^{(n)}$ denotes the approximation for L_{ir} in the nth step of iteration.

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