Optimum Partition of Bandwidth Contingents

Paul Schlüter and Joachim Charzinski
ICN M NT 6, Siemens AG, München

SIEMENS

Contents

A given bandwidth must be shared by upstream and downstream traffic.

Sometimes, individual limits must be imposed.

What is the best way to do this? ...

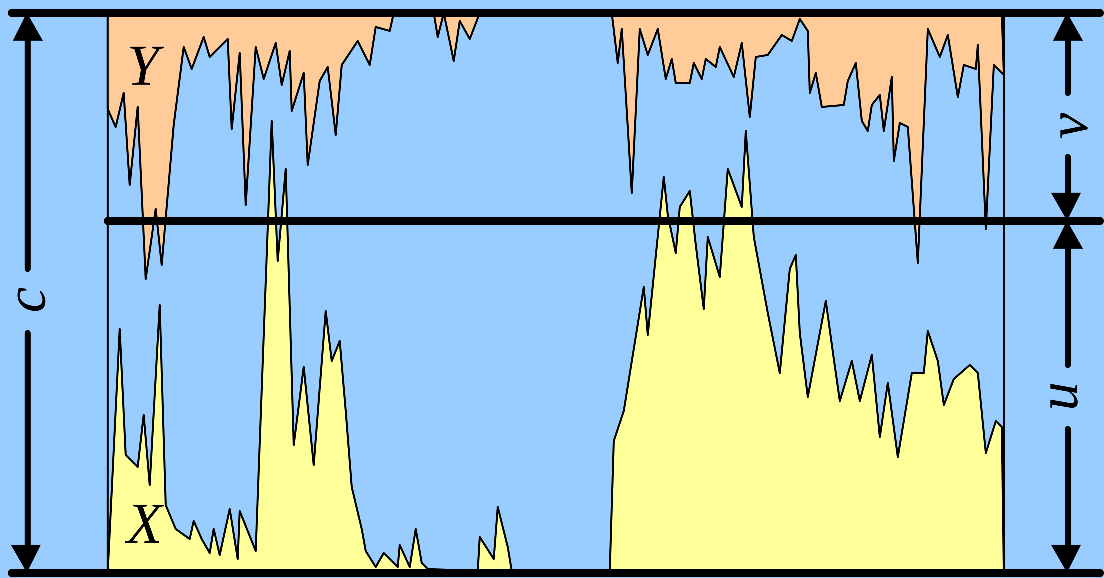
- Rate Limitations
  shared limit, individual limits
- Loss Function
  definition, properties
- Normally Distributed Rates
- Measured Rates
- Conclusions and Next Steps

Rate Limitations:

Shared Limit vs. Optimum Individual Limits

A given total bandwidth c is to be used by two (or more) traffic sources X, Y, ... There can be

- a shared limit: $X+Y < c$
- individual limits: $X < u$ and $Y < v$ for some static $u+v = c$.



Rate Limitations:

Relevance

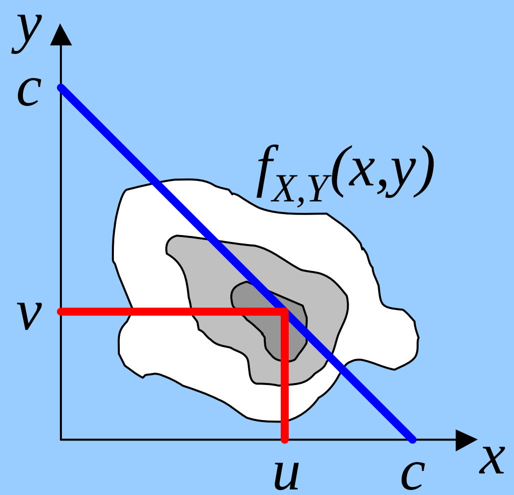
Individual limits (with X, Y = up, down rate) are important for

- Powerline Communication
- ADSL
- Hiperlan
- UMTS, TD-CDMA mode

- What is the optimum partition $u_o + v_o = c$?
- What can be gained by a dynamic adjustment of u and v?
  - time scale?
  - algorithms?

Loss Function:

Definition



shared limit

Heaviside f. expectation

$$d(c) = \frac{1}{m_X + m_Y} \left\langle (X + Y - c) \cdot H(X + Y - c) \right\rangle$$

individual limits

$$b(u,v) = \frac{1}{m_X + m_Y} \left\langle (X - u) \cdot H(X - u) + (Y - v) \cdot H(Y - v) \right\rangle$$

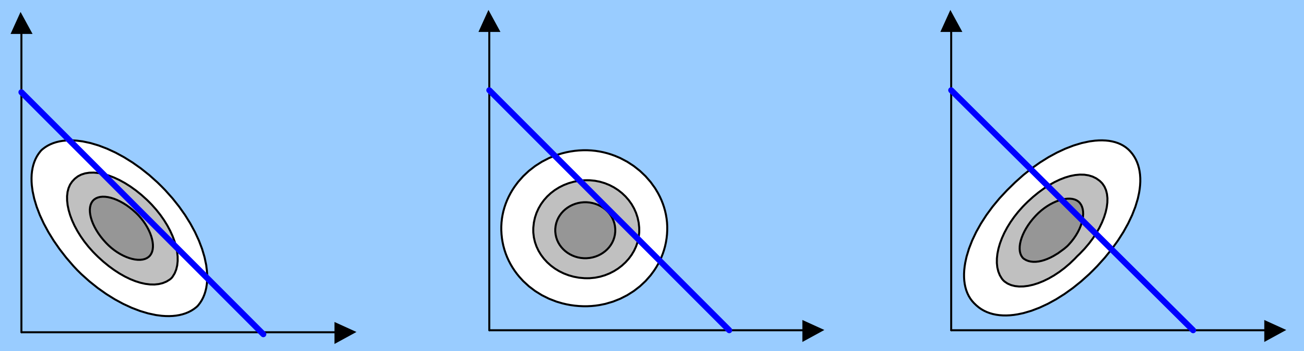
$$b(c) = b(u_o, v_o) = \min_{u+v=c} b(u,v) \geq d(c)$$

Loss Function:

Shared Limit

$$d(c) = \frac{1}{m_X + m_Y} \int_c^\infty dz (z-c) f_{X+Y}(z)$$

- depends on density of $X+Y$ only
- increases with increasing correlation coefficient r (if marginal densities of X and Y are fixed)



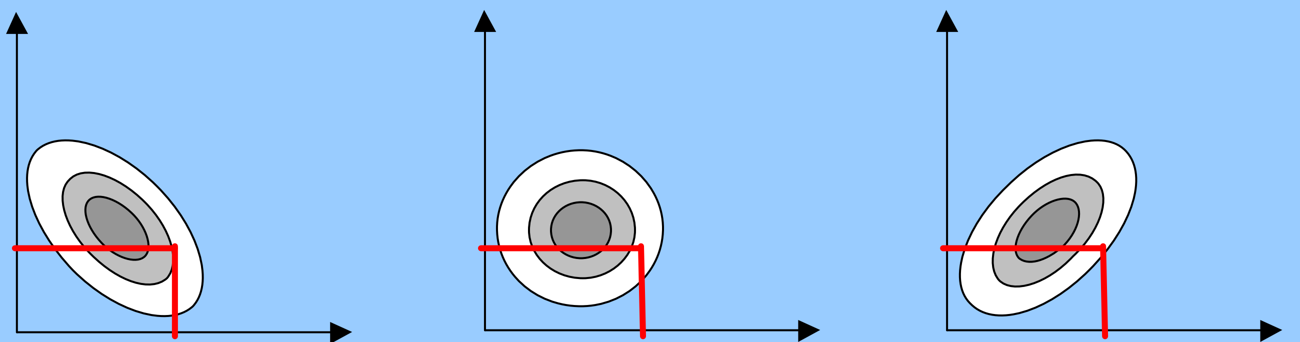
$$d(c; r<0) \quad \leq \quad d(c; r=0) \quad \leq \quad d(c; r>0)$$

Loss Function:

Individual Limits

$$b(u,v) = \frac{1}{m_X + m_Y}\left(\int_u^\infty dx(x-u) f_X(x) + \int_v^\infty dy(y-v) f_Y(y)\right)$$

- depends on marginal densities of X and Y only
- is independent of correlation coefficient r (if marginal densities of X and Y are fixed)



$$b(u,v;\, r<0) \quad = \quad b(u,v;\, r=0) \quad = \quad b(u,v;\, r>0)$$

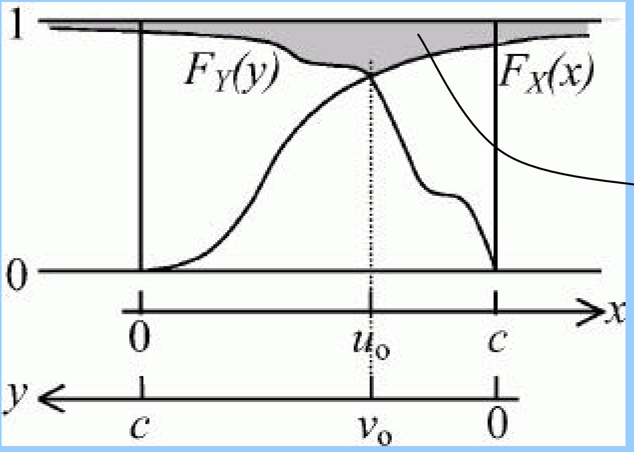
Loss Function:

Optimum Individual Limits

The optimum partition (u_o, v_o) is given by

$$u_o + v_o = c$$

$$F_X(u_o) = F_Y(v_o)$$



loss $(m_X + m_Y)b(c)$

- in the optimum, both limits are violated with the same probability $1\text{-}F_X(u_o) = 1\text{-}F_Y(v_o)$
- losses in X and Y normally different

Normally Distributed Rates:

Shared Limit

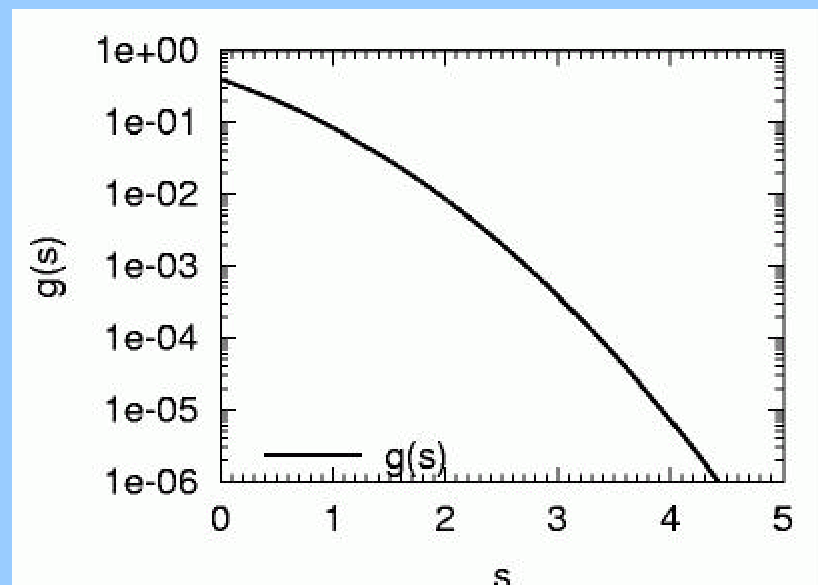
Density of $X+Y$ is completely defined by

$$m_{X+Y} = m_X + m_Y$$

$$s_{X+Y} = \sqrt{s_X^2 + 2 r s_X s_Y + s_Y^2}$$

loss

$$d(c) = \frac{s_{X+Y}}{m_{X+Y}} g\left(\frac{c - m_{X+Y}}{s_{X+Y}}\right)$$

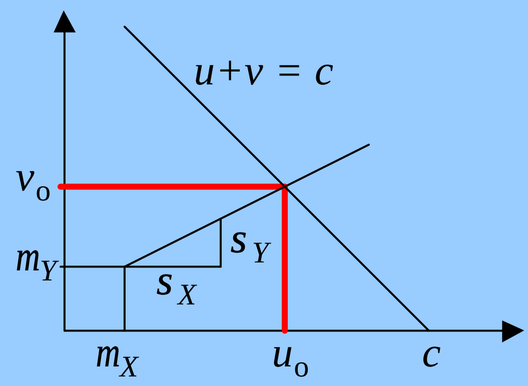


Normally Distributed Rates:

Optimum Individual Limits

Marginal densities of X and Y completely defined by m_X, m_Y, s_X, s_Y.

u_o and v_o



optimum loss

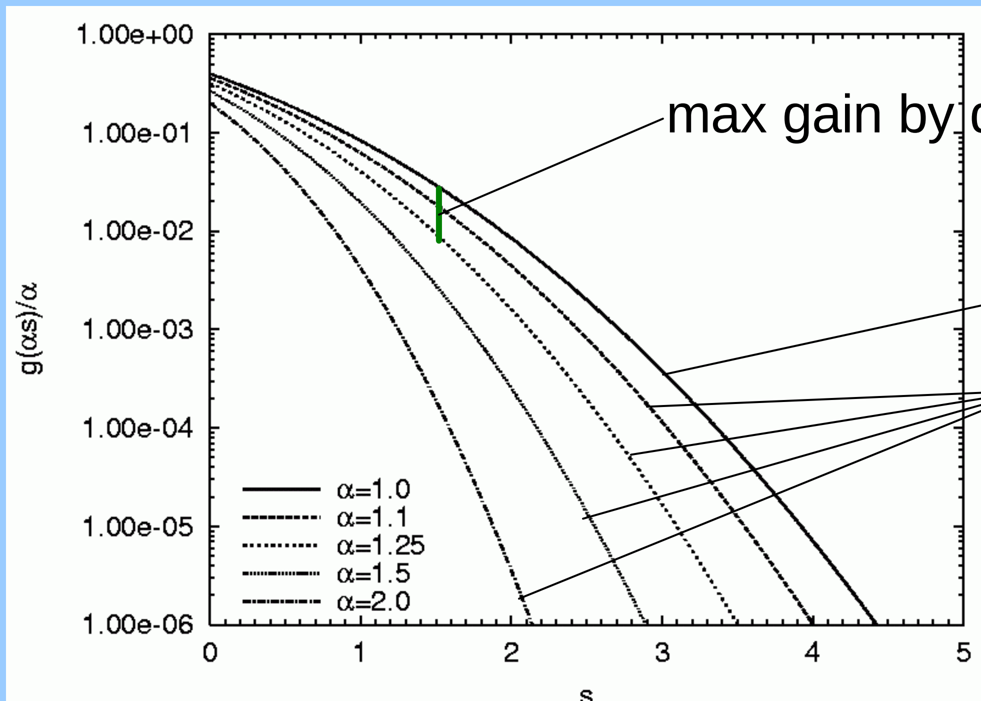
$$b(c) = b(u_o, v_o) = \frac{s_X + s_Y}{m_X + m_Y}\, g\left(\frac{c - (m_X + m_Y)}{s_X + s_Y}\right) = d(c; r = 1)$$

width of optimum

$$|\Delta u| = |\Delta v| \approx \sqrt{2 s_X s_Y} \quad \text{for } c \approx m_X + m_Y \quad (\propto 1/c \ \text{ for } c \to \infty)$$

Normally Distributed Rates:

Dynamic Adjustment

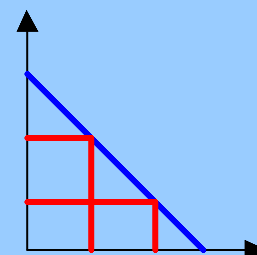


max gain by dynamic adjustment

$b\ d(c; r = 1) = b\ b(c)$

$b\ d(c; r < 1)$

$$\left(a = \frac{s_X + s_Y}{s_{X+Y}} \geq 1 \right)$$



dynamic adjustment

- ineffective if loss is large ($\alpha \approx 1.1$)
- effective for negatively correlated X, Y and s_X » s_Y.

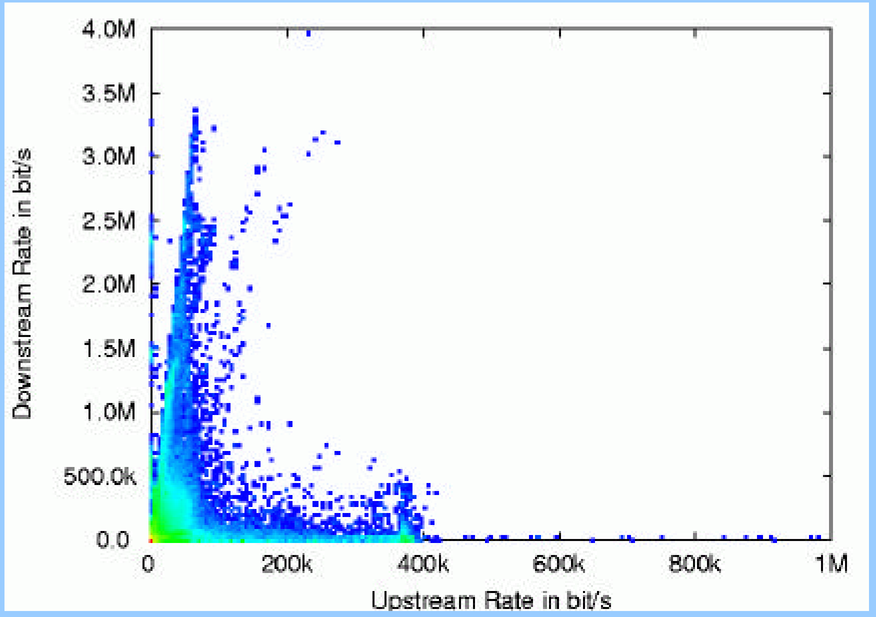
Measured Rates:

Overview (1)

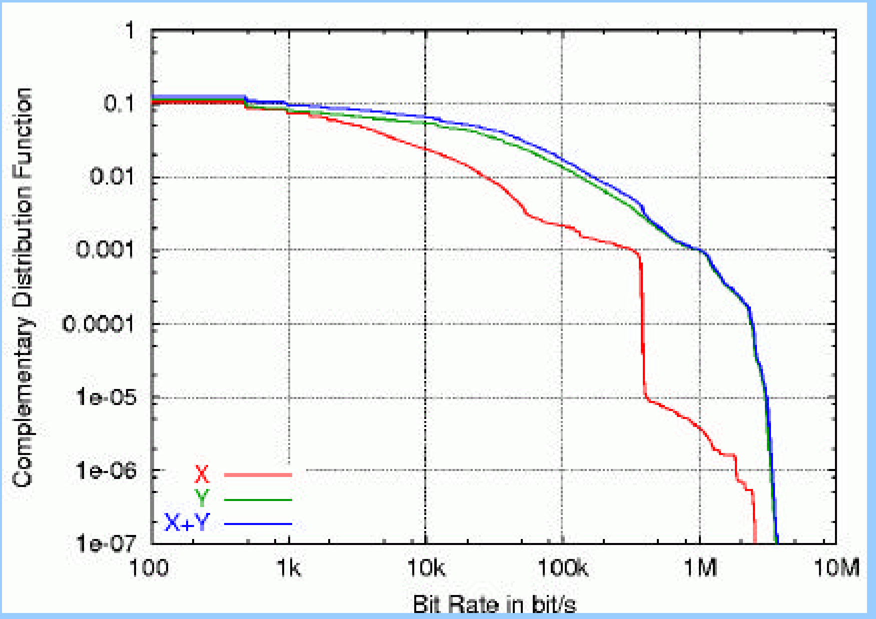
- data from ADSL trial, Münster, 1998
- all TCP and UDP packets except those for video
- concentrated traffic from ≈ 7 users
- 1s time slots
- summary characteristics
  up: $m_X = 1.4$ kb/s, $s_X = 13$ kb/s,
  down: $m_Y = 6.2$ kb/s, $s_Y = 71$ kb/s,
  up/down: $r = 0.23$
- definitely not normally distributed

Measured Rates:

Overview (2)



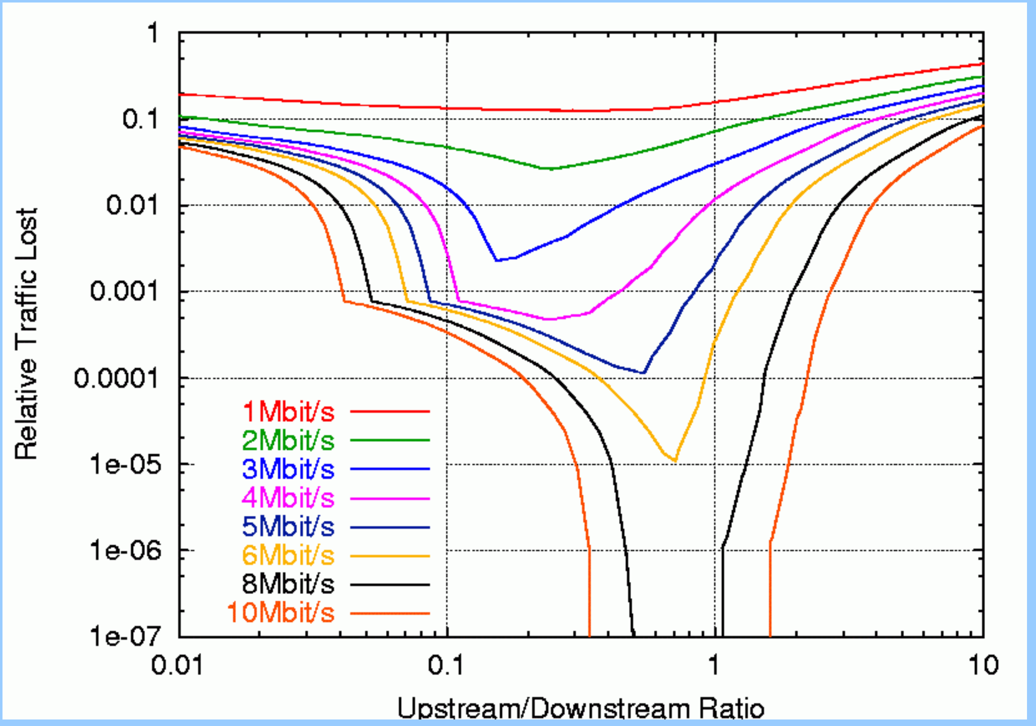
joint density



complementary distribution

Measured Rates:

Individual Limits



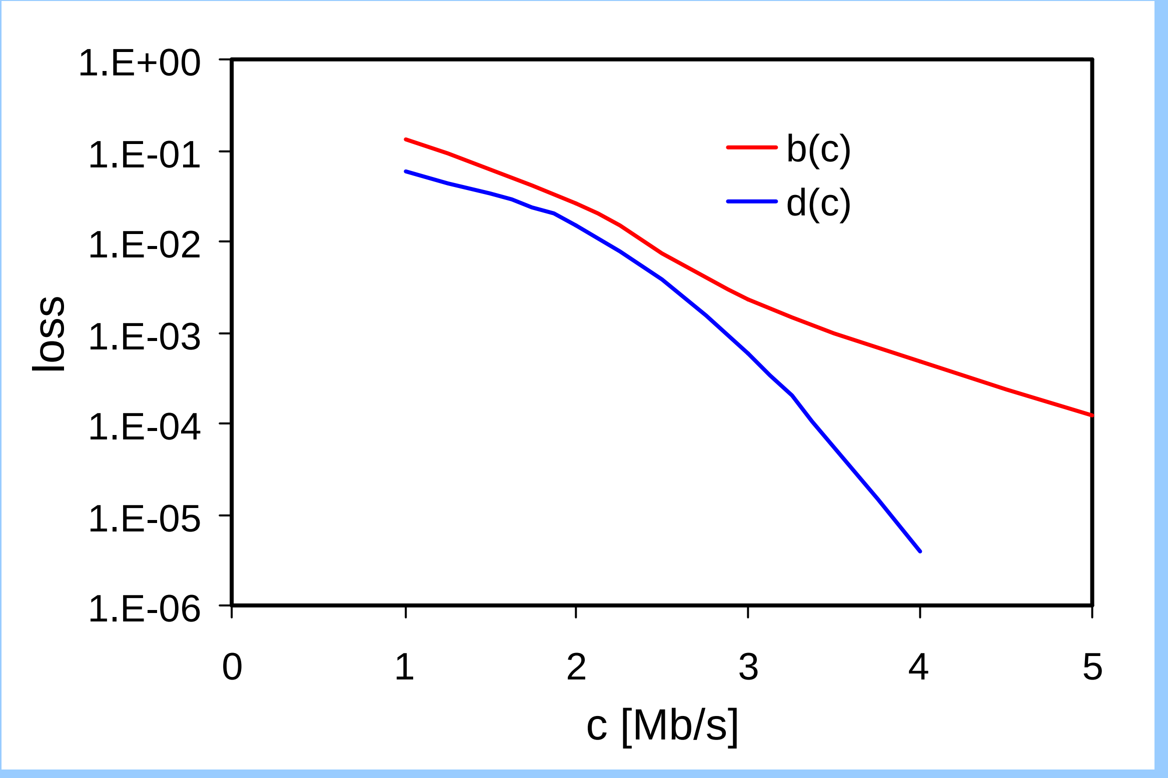
Optimum

- very broad for small $u+v = c$ (large loss)
- deep and narrow for large $u+v = c$

SIEMENS

Measured Rates:

Dynamic Adjustment



Dynamic adjustment rather ineffective for small c (large loss).

Conclusions and Next Steps

- knowledge of joint density of X and Y not necessary; (marginal) densities of X, Y, and $X+Y$ suffice
- quantitative aspects most easily understood by study of normally distributed rates
- for total bandwidth close to sum of means (c » $m_X + m_Y$)
  - loss large (> 10%)
  - optimum of individual limits broad
  - small gain by dynamic adjustment
- for total bandwidth large
  - loss arbitrarily small
  - optimum of individual limits narrow
  - large gain by dynamic adjustment (imaginable)
- next steps: time scale, algorithms for dynamic adjustments, ...