Optimum Partition of Bandwidth Contingents

Paul Schlüter and Joachim Charzinski
ICN M NT 6, Siemens AG, München

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A given bandwidth must be shared by upstream and downstream traffic.
Sometimes, individual limits must be imposed.
What is the best way to do this? ...

- Rate Limitations
  shared limit, individual limits
- Loss Function
  definition, properties
- Normally Distributed Rates
- Measured Rates
- Conclusions and Next Steps
Rate Limitations:

Shared Limit vs. Optimum Individual Limits

A given total bandwidth $c$ is to be used by two (or more) traffic sources $X$, $Y$, ... There can be

- a shared limit: $X + Y < c$
- individual limits: $X < u$ and $Y < v$ for some static $u + v = c$. 
Rate Limitations:

Relevance

Individual limits (with $X$, $Y = \text{up}, \text{down rate}$) are important for
- Powerline Communication
- ADSL
- Hiperlan
- UMTS, TD-CDMA mode

- What is the optimum partition $u_o + v_o = c$?
- What can be gained by a dynamic adjustment of $u$ and $v$?
  - time scale?
  - algorithms?
Loss Function:

Definition

shared limit

\[ d(c) = \frac{1}{\mu_x + \mu_y} \langle (X + Y - c) \cdot H(X + Y - c) \rangle \]

individual limits

\[ b(u, v) = \frac{1}{\mu_x + \mu_y} \langle (X - u) \cdot H(X - u) + (Y - v) \cdot H(Y - v) \rangle \]

\[ b(c) = b(u_0, v_0) = \min_{u+v=c} b(u, v) \geq d(c) \]
Loss Function:

Shared Limit

\[ d(c) = \frac{1}{\mu_X + \mu_Y} \int_c^\infty dz (z - c) f_{X+Y}(z) \]

- depends on density of \(X+Y\) only
- increases with increasing correlation coefficient \(\rho\) (if marginal densities of \(X\) and \(Y\) are fixed)

\[ d(c; \rho < 0) \leq d(c; \rho = 0) \leq d(c; \rho > 0) \]
Loss Function:

Individual Limits

\[ b(u, v) = \frac{1}{\mu_X + \mu_Y} \left( \int_{-\infty}^\infty dx(x-u)f_X(x) + \int_{-\infty}^\infty dy(y-v)f_Y(y) \right) \]

- depends on marginal densities of \( X \) and \( Y \) only
- is independent of correlation coefficient \( \rho \) (if marginal densities of \( X \) and \( Y \) are fixed)

\[ b(u, v; \rho < 0) = b(u, v; \rho = 0) = b(u, v; \rho > 0) \]
Loss Function:

Optimum Individual Limits

The optimum partition \((u_o, v_o)\) is given by

\[
\begin{align*}
    u_o + v_o &= c \\
    F_X(u_o) &= F_Y(v_o)
\end{align*}
\]

- In the optimum, both limits are violated with the same probability
  \(1 - F_X(u_o) = 1 - F_Y(v_o)\)
- Losses in \(X\) and \(Y\) normally different

\[
\text{loss} \ (\mu_X + \mu_Y)b(c)
\]
Normally Distributed Rates:

Shared Limit

Density of $X+Y$ is completely defined by

$$\mu_{X+Y} = \mu_X + \mu_Y$$

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + 2\rho\sigma_X\sigma_Y + \sigma_Y^2}$$

loss

$$d(c) = \frac{\sigma_{X+Y}}{\mu_{X+Y}} g\left(\frac{c - \mu_{X+Y}}{\sigma_{X+Y}}\right)$$
Normally Distributed Rates:

Optimum Individual Limits

Marginal densities of $X$ and $Y$ completely defined by $\mu_X$, $\mu_Y$, $\sigma_X$, $\sigma_Y$.

$u_0$ and $v_0$

\[ u_0 + v_0 = c \]

optimum loss

\[ b(c) = b(u_0, v_0) = \frac{\sigma_X + \sigma_Y}{\mu_X + \mu_Y} g \left( c - \frac{\mu_X + \mu_Y}{\sigma_X + \sigma_Y} \right) = d(c; \rho = 1) \]

width of optimum

\[ |\Delta u| = |\Delta v| \approx \sqrt{2\sigma_X \sigma_Y} \text{ for } c \approx \mu_X + \mu_Y \quad (\propto \frac{1}{c} \text{ for } c \to \infty) \]
Normally Distributed Rates:
Dynamic Adjustment

\[ r_{b_{\text{cd}}} = \max \text{gain by dynamic adjustment} \]

\[ \beta d(c; \rho = 1) = \beta b(c) \]

\[ \beta d(c; \rho < 1) \]

\[
\alpha = \frac{\sigma_X + \sigma_Y}{\sigma_{X+Y}} \geq 1
\]

dynamic adjustment
- ineffective if loss is large (\( \alpha \approx 1.1 \))
- effective for negatively correlated \( X, Y \) and \( \sigma_X \approx \sigma_Y \).
Measured Rates:

Overview (1)

- data from ADSL trial, Münster, 1998
- all TCP and UDP packets except those for video
- concentrated traffic from ≈ 7 users
- 1s time slots
- summary characteristics
  up: \( \mu_X = 1.4 \text{ kb/s}, \sigma_X = 13 \text{ kb/s} \),
  down: \( \mu_Y = 6.2 \text{ kb/s}, \sigma_Y = 71 \text{ kb/s} \),
  up/down: \( \rho = 0.23 \)
- definitely not normally distributed
Measured Rates: Overview (2)

joint density  complementary distribution
Measured Rates:

Individual Limits

Optimum

- very broad for small $u+v = c$ (large loss)
- deep and narrow for large $u+v = c$
Measured Rates:

Dynamic Adjustment

Dynamic adjustment rather ineffective for small $c$ (large loss).
Conclusions and Next Steps

- Knowledge of joint density of $X$ and $Y$ not necessary; (marginal) densities of $X$, $Y$, and $X+Y$ suffice
- Quantitative aspects most easily understood by study of normally distributed rates
- For total bandwidth close to sum of means ($c \approx \mu_X + \mu_Y$)
  - Loss large (> 10%)
  - Optimum of individual limits broad
  - Small gain by dynamic adjustment
- For total bandwidth large
  - Loss arbitrarily small
  - Optimum of individual limits narrow
  - Large gain by dynamic adjustment (imaginable)

- Next steps: time scale, algorithms for dynamic adjustments, ...