Wuerzburger Workshop "IP Management, IP Netzplanung und Optimierung"

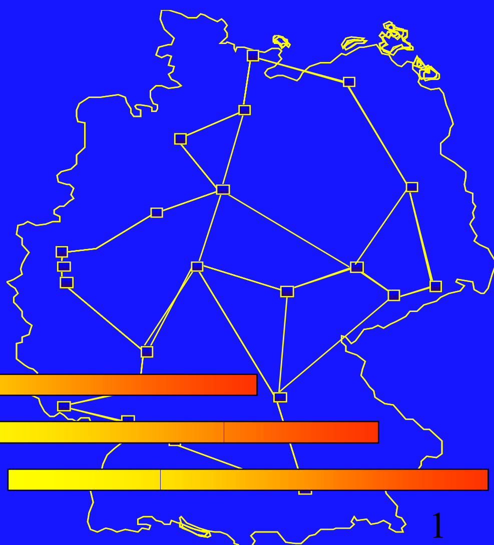
Lehrstuhl für Informatik III, Universität Würzburg, 27./28.07.2004

Multi-rate Queueing Loss and Processor Sharing Systems
A brief but incomplete Summary of recent Results

Harro L. Hartmann

E-mail: H. Hartmann@tu-bs.de

Institute of Computer Technology and Communication Networks

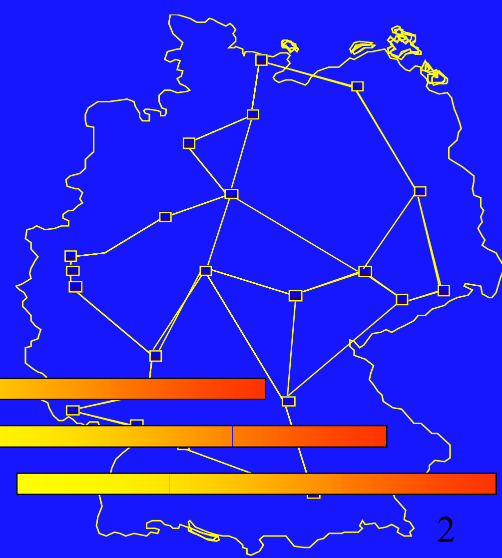


D
C
N

Department of **C**ommunication **N**etworks (DCN)

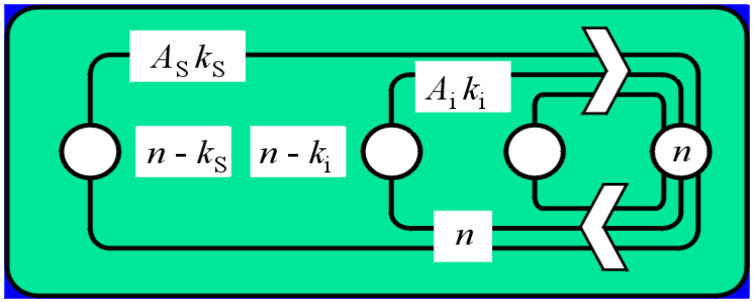
Multi-rate Queueing Loss and Processor Sharing Systems
A brief but incomplete Summary of recent Results

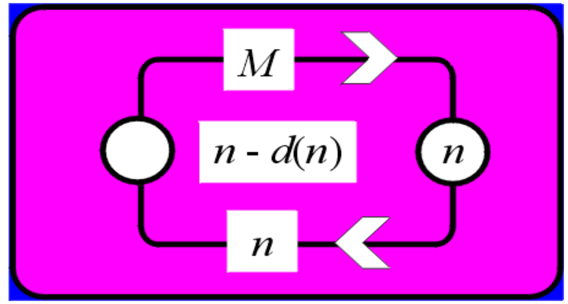
- Basic Assumptions and Objectives
- The One-Level Equivalence of the Multi-level Kaufman Roberts Recurrence Equations for buffer-less Systems, Conclusions I
- Generalization of the Multi-rate Recurrence Equations with respect to Queueing Loss Systems
- Applications of the generalized One-level Functional Equation to Multi-rate Processor Sharing Systems, Conclusions II



Department of **C**ommunication **N**etworks (DCN)

Service type $1 \leq i \leq S$ modulated Poisson arrivals of specified service distribution but a common expected mean rate μ □□□ state. Local balance equations (*micro-state interrelations omitted*):





Ka-Ro multi rate difference equation (DE):

$$n \cdot p_n = \sum_{i=1}^{S} A_i \cdot k_i \cdot p_{n-k_i} = M \cdot \sum_{i=1}^{S} sp_i \cdot p_{n-k_i} , \quad \sum_{i=1}^{S} sp_i = 1 \; .$$

Suggested one-level functional equation (FE):

$$n \cdot \hat{p}_n = M \cdot \hat{p}_{n-d(n)} \,, \quad M = A \cdot d(n).$$

Problem definition: $\hat{p}_n \cong p_n$ for the entire state space subject to (s.t.) $d(n, M, \boldsymbol{P}, \boldsymbol{K}) \approx m_{1K}$.

Functional equations (FEs) interrelate functions and operators on a common set of numbers. Special FE-types are ***difference equations*** (DEs), differential-, integral- equations, etc.

$$i! = i \cdot (i-1)! \qquad i \cdot p_i = A \cdot p_{i-1} \qquad \Gamma(x+1) = x \cdot \Gamma(x) \qquad x \cdot f(x) = A \cdot f(x-1)$$

$$P_i(A) = \frac{A^i}{i!} e^{-A} \qquad F_G(x, A) = \frac{\int_A^\infty t^x e^{-t} dt}{\Gamma(1+x)} \equiv \frac{\Gamma(1+x, A)}{\Gamma(1+x)} \qquad f(x) = D \frac{A^x}{\Gamma(1+x)}$$

Solution of the multi-level Recurrence- by a one-level Functional-Equation for bufferless Systems

Original Domain | **Transform Domain**

$$n \cdot p_n = \sum_{i=1}^{S} A_i \cdot k_i\, p_{n-k_i}$$

$$\chi_W(\mathrm{j}\omega) = \exp\left[\sum_{i=1}^{S} A_i\left(\mathrm{e}^{\mathrm{j}\omega \cdot k_i} - 1\right)\right]$$

J. Y. Hui 90

$$b_i = P(> C - k_i) = \sum_{n > C - k_i}^{C} p_n$$

$$d(M, n, \boldsymbol{K}, \boldsymbol{P}) \approx m_{1K}\left[1 + \frac{cv_K^2}{\gamma}\left(1 - \frac{M}{n}\right)\right],\ \gamma = \begin{cases} 3, & n > M, \\ 2, & n \le M. \end{cases}$$

$$n \cdot \hat{p}_n = M \cdot \hat{p}_{n-d}$$

$$\chi_R(\mathrm{j}\omega, d) = \exp\left[\frac{M}{d}\left(\mathrm{e}^{\mathrm{j}\omega \cdot d} - 1\right)\right]$$

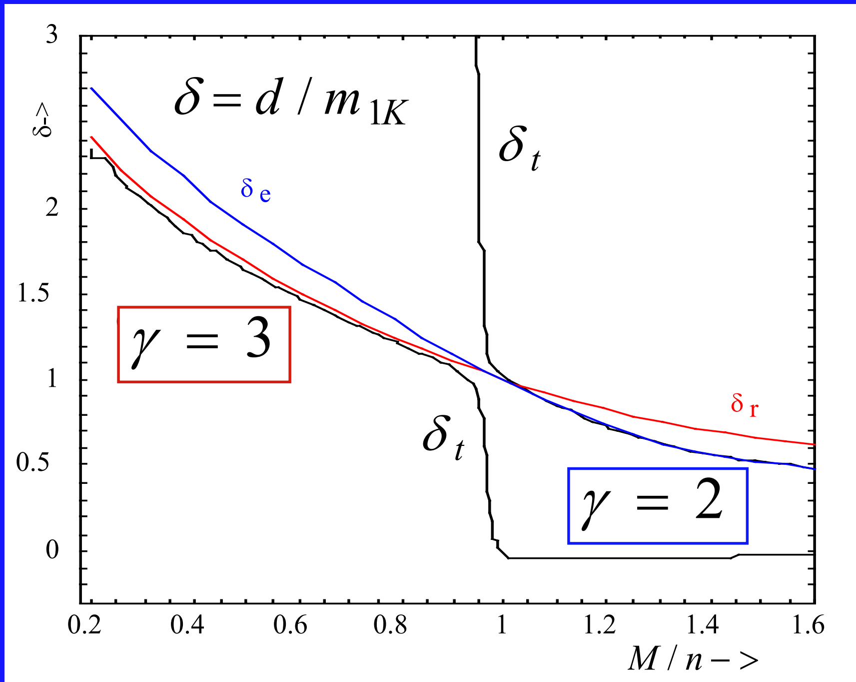
$$\hat{p}_n \cong \frac{1 - (n/M)^{1/d}}{1 - (n/M)} E\left(\frac{n}{d}, \frac{C}{d}, \frac{M}{d}\right) \approx \frac{1}{d} E\left(\frac{n}{d}, \frac{C}{d}, \frac{M}{d}\right)$$

$$\hat{b}_i = \int_{C-k_i}^{C} \hat{p}_n\, dn \cong \frac{1 - (C/M)^{k_i/d}}{1 - (C/M)} E1\left(\frac{C}{d}, \frac{M}{d}\right) \approx \frac{k_i}{d} E1\left(\frac{C}{d}, \frac{M}{d}\right)$$

$$Y = M \cdot \left[1 - E(B_i)\right] \cong M \cdot \left[1 - E1\left(\frac{C}{d}, \frac{M}{d}\right)\right]$$

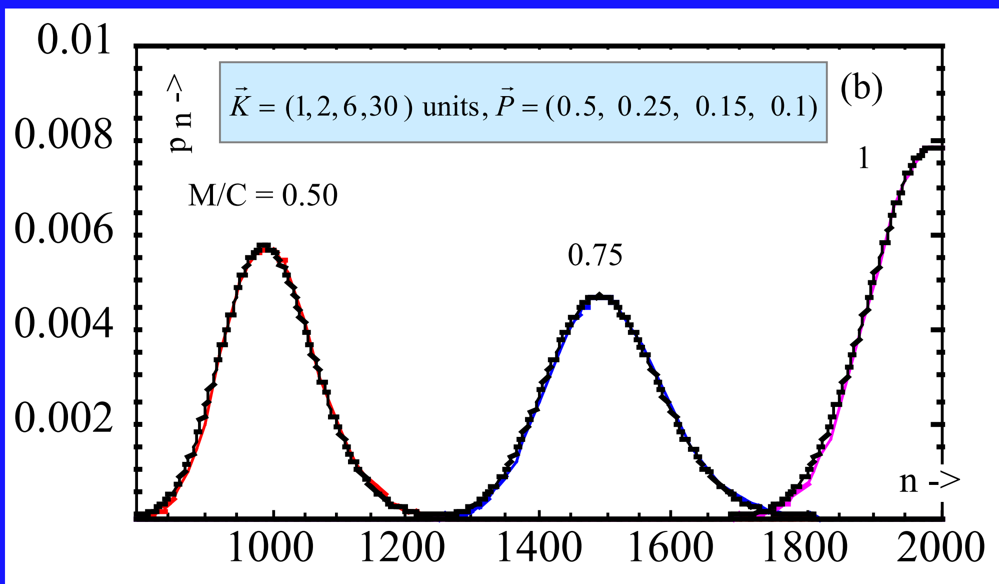
Erl Mb/s carried

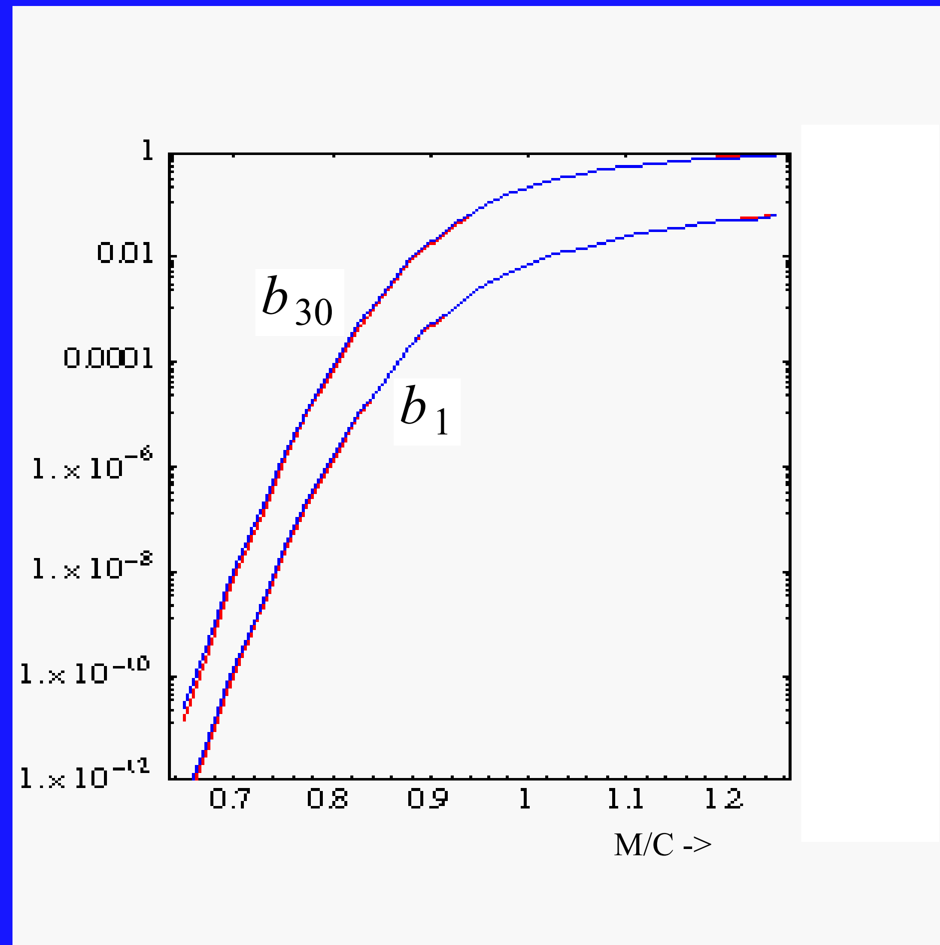
***Result*: Near-explicit continuous solutions covering the entire State and Service Range.**



Tangents at $M \approx n$:

$$d(n, M, K, P) \approx m_{1K}\left[1 + \frac{c v_K^2}{\gamma}\left(1 - \frac{M}{n}\right)\right], \quad \gamma = \begin{cases} 3, & M < n, \\ 2, & M \geq n. \end{cases}$$





Extracted Conclusions I

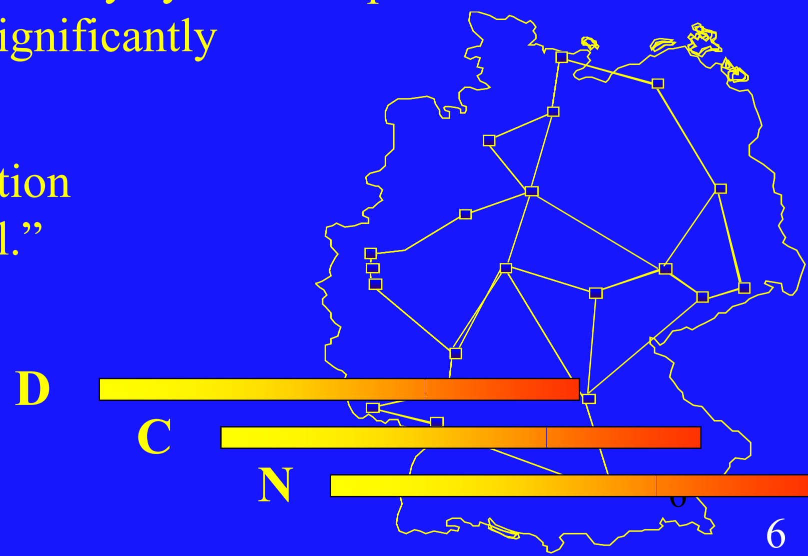
Effective recurrence depth $k_1 < d < k_S$ equals effective bit rate of the bursty ensemble stream under progress at each state n or bit rate of the state-equivalent one-level system.

Peaked single class stream $z_i = d$ -> Fredericks-Haywards blocking approximation $E1(C/d, M/d)$ = expected value of B.

Scaling features: If for fixed $\boldsymbol{P}$, $\boldsymbol{K}_c = c\,\boldsymbol{K}$, then $d_c = c\,d$. If furthermore c commonly scales C and M, all results remain unchanged.

Limitations: The parameter identification of d in the MGF transform domain looses phase information used for the one-level CHF inverse, e. g. probability cycles of sophisticated ($\boldsymbol{P}$,$\boldsymbol{K}$) combinations are averaged. d must not change significantly within $[n - d, n]$.

Advantages:"Inherent elegance of the continuous FE-solution together with the fact that the approximation error is small."
Applications to multi-rate traffic NHR cf. reference)*.
Generalization to multi-rate queueing systems to be considered now.



)*H. L. Hartmann, ITC 17, 2001, Salvador da Bahia (Brazil).

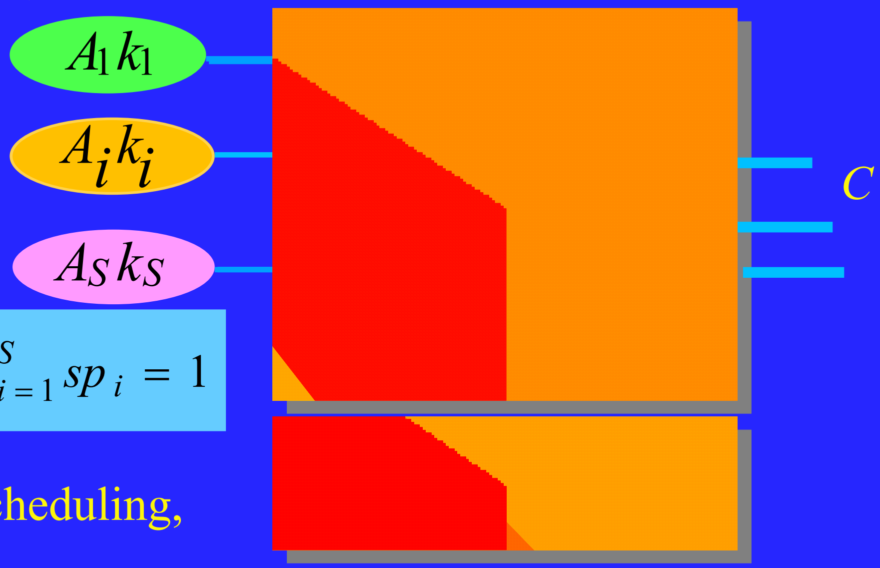
The GI modulated M/M/C/C_b Queueing Loss and M/G/C Processor Sharing System

Multi-rate buffered switch with selected backbone links of capacity *C* performing the complete sharing mode for the total demand *M*.

Committed modem rates k_i of service type *i* with traffic demand A_i. Finite number of service types *S* or truncated pdf of bandwidth.

Poisson arrivals of general distributed amplitudes s. t.

$$M = \sum_{i=1}^{S} A_i k_i$$



General independent distributed bit rates or truncated bit rate densities.

$$sp_i = \frac{A_i k_i}{M}, \quad \sum_{i=1}^{S} sp_i = 1$$

Arbitrary switch fabric aside selected queue scheduling, queue service & route processing.

Process interrelations verified by the stationary state probability density functions (pdfs) and conditional test customers response times.

Generalization of the Ka-Ro Recurrence to States beyond C, e.g. $C + C_b$.

Original Domain | **Transform Domain**

$$Min[n,C]\cdot p_n = M\cdot\sum_{i=1}^{S} s p_i\, p_{n-k_i},\quad n\le C+C_b$$

$$b_i = P\{W > C + C_b - k_i\}$$

$$\chi_W(s) = \chi_W^{(P)}(s) - P(\ge C)e^{sC} + \frac{p_C\cdot e^{sC}}{1-\frac{1}{C}\sum_{i=1}^{S} A_i k_i e^{sk_i}},$$

$$d(M,n,\boldsymbol{K},\boldsymbol{P}) \cong m_{1K}\Big[1+\frac{cv_K^2}{\gamma}\Big(1-\frac{M}{n}\Big)\Big],\quad \gamma=\begin{cases}3, & n<C,\\ 2, & n\ge C.\end{cases}$$

Functional Equation of the pdf

$$Min[n,C]\cdot\hat{p}_n = M\cdot\hat{p}_{n-d}$$

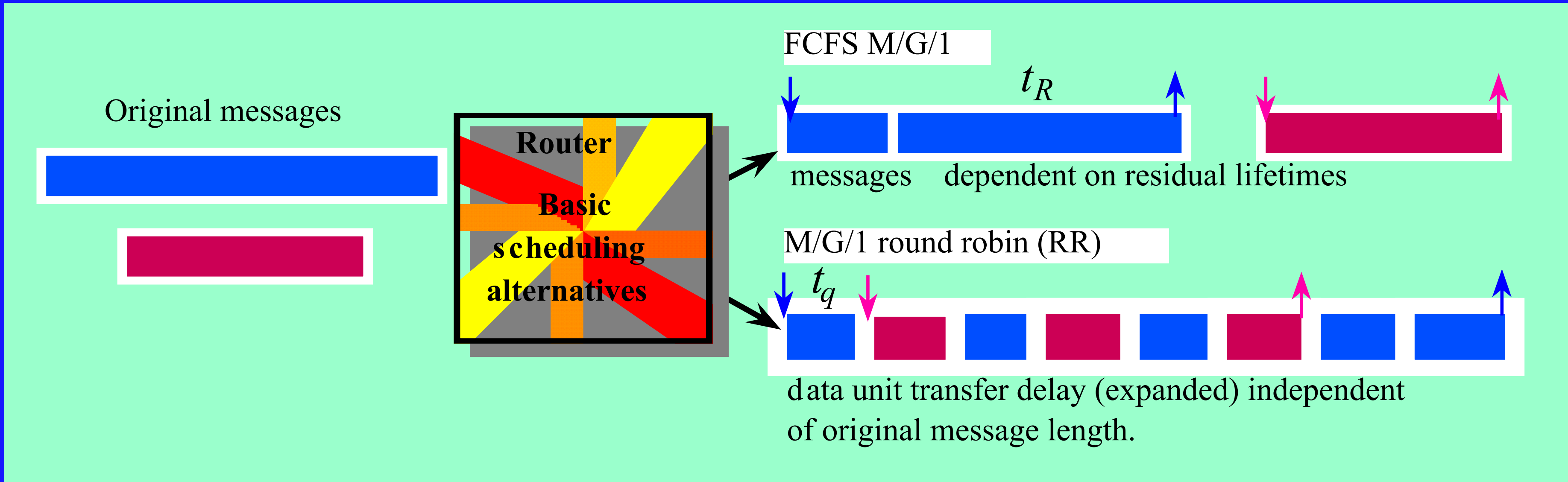
$$\chi_R(s) = \chi_R^{(P)}(s) - \hat{P}(W\ge C)e^{sC} + \frac{\hat{p}_C e^{sC}}{1-\frac{M}{C}e^{sd}}$$

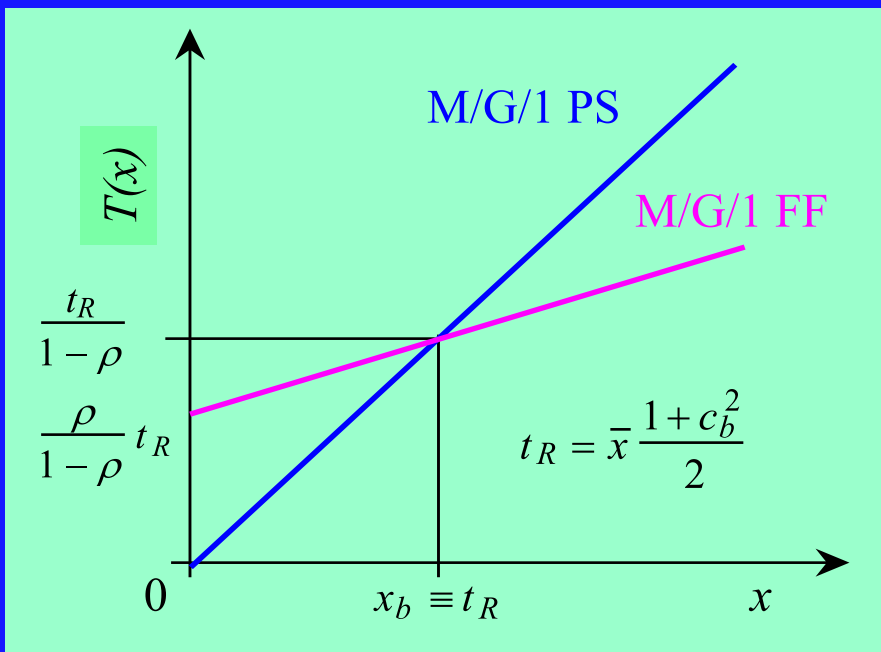
$$\hat{p}_n = \frac{D}{d}e^{-A}\begin{cases}\dfrac{A^{n/d}}{\Gamma(1+n/d)}, & 0\le n\le C,\\[4pt] & A = M/d\\[4pt] \dfrac{A^{C/d}\rho^{(n-C)/d}}{\Gamma(1+C/d)}, & C<n\le\begin{cases}\infty,\\ C+C_b\end{cases}\end{cases}$$

$$T = \int_0^\infty T(x)b(x)\,dx \equiv \frac{E[n]}{d\cdot\lambda} = \frac{\chi_R^{(1)}(0)}{d\cdot\lambda}$$

$$T(x_t) \cong x_t\frac{k_t}{m_{1K}}\left[1+\frac{E2[N,\rho N]}{N(1-\rho)}\right]\qquad \text{(M/M/N FF)}\quad N\cong C/m_{1K}$$

***Result*: Conditional response time of a test customer with message duration x_t and bit rate k_t**

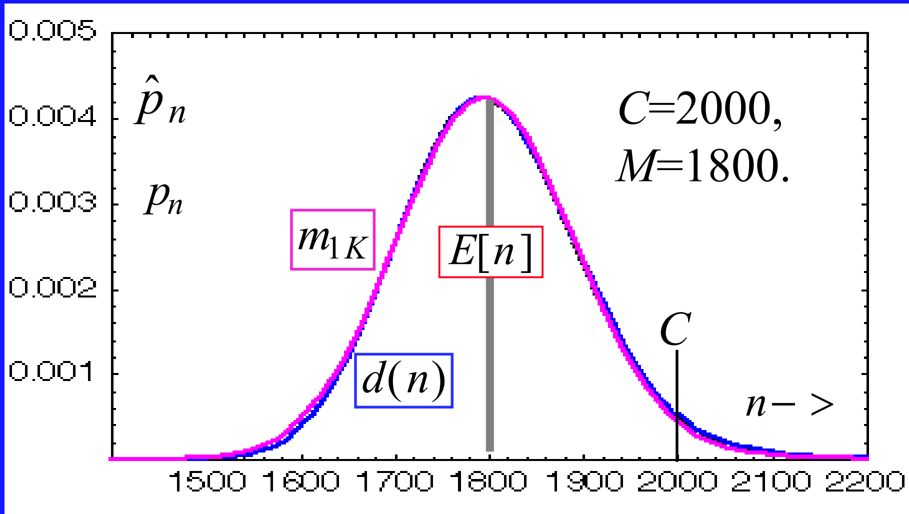


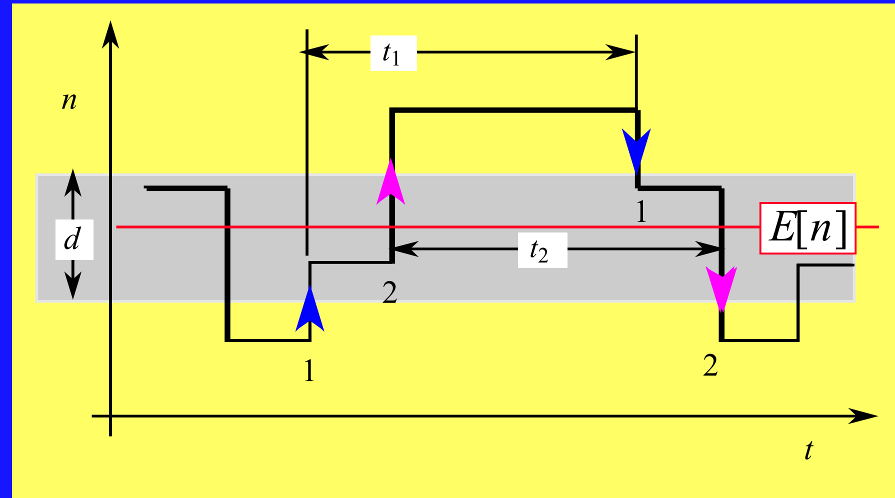


$$T(x) = E\{[X_t + W(X_t)] \mid X_t = x\}, \quad T = E[T(x)]$$

$$T = \int_0^\infty T(x)\, b(x)\, dx \equiv \frac{E[n]}{d \cdot \lambda} \cong \frac{\chi_R^{(1)}(0)}{m_{1K} \cdot \lambda}$$

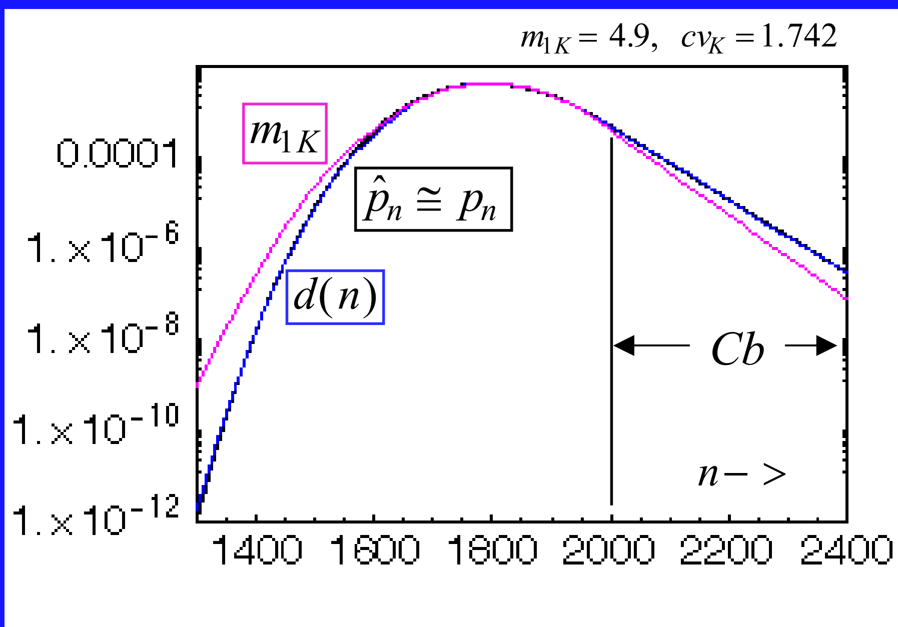
With heavy tailed service time distributions, the breakpoint x_b increases by c_b. Thus, processor sharing (PS) response remains fair and more efficient than feedforward (FF) queue service.

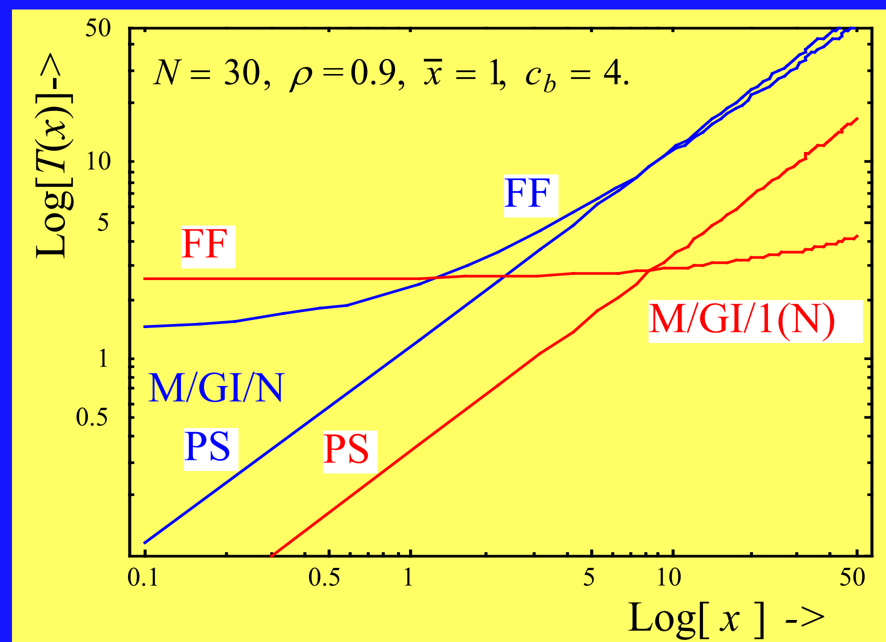




$$\boldsymbol{K} = (1, 2, 6, 30) \text{ units}, \quad \boldsymbol{P} = (0.5,\ 0.25,\ 0.15,\ 0.1)$$

$$E[n] = \int_0^\infty n \cdot \hat{p}_n dn = d \cdot \lambda \cdot T = M \cdot f_D$$





Open Loop Streaming**:** It avoids download the entire file (e.g. audio, video, real time and playback). Thus, a client can playout the file *without excessive delay*. 1999 K. Lindberger)* proposed a flow assignment (FA) for stream traffic based on mean effective bandwidths *e* and mean peak bit rates *h* of the multi-rate ensemble. Our equivalent blocking relation reads in this almost "bufferless" case:

$$b_h \cong \frac{1-(C/M)^{h/e}}{1-C/M} E1\left(\frac{C}{e}, \frac{M}{e}\right) \approx \frac{h}{e} E1\left(\frac{C}{e}, \frac{M}{e}\right)^* = \text{GoS}$$

Now, FA determines the admissable M. But *e* and *h* are defined according to *d*. For peak rate admission $e = h = d$ and $b_d = E(B) = E1$. Note e.g. Lindberger's $e/h = 0.45$.

Closed Loop Elastic Applications**:** Elastic traffic (e.g. electronic mail, file transfer, remote access, and Web transfers) can make use of *as much or as little bandwidth* as happens to be available. Of course, the more bandwidth, the better. Therefore buffered fair peak rate orientation for heavy tailed documents are favored. Low response times s.t. QoS and very low target blockings may be achieved by M/G/1(N) *complete* PS (CPS) systems instead of their *partial* PS (PPS) forerunners.

$$T(x_t) = \frac{x_t \cdot k_t}{m_{1K}} f_D, \quad f_D = \left[1 + \frac{E2(N, \rho \cdot N)}{N(1-\rho)}\right]^*, \; N \cong \frac{C}{m_{1K}}, \; \rho = \frac{M}{C} < 1 \; .$$

M/M/N FF or M/G/N PPS.

$$T(x_t) = \frac{x_t \cdot k_t}{m_{1K}} f_D, \quad f_D = \frac{1}{N}\left[1 + \frac{\rho}{1-\rho}\right] = \frac{m_{1K}}{C(1-\rho)} = \frac{1}{N(1-\rho)} \; .$$

M/M/1(N) FF or M/G/1(N) CPS.

Extracted Conclusions II

The extended Ka-Ro multi-level equation system were iteratively solved. This enumeration requires CPU times of $O[S(C + C_b)]$ and provides accurate discrete state probabilities. Unfortunately, this complexity occurs again and again if the parameter set (M, $\boldsymbol{K}$, $\boldsymbol{P}$) changes.

The investigated one-level state-equivalent functional equation covers the entire continuous state space. It provides a *near explicit solution of optional accuracy*. In order to maintain time complexities $O(1)$ per state percentage inaccuracies due to fast normalization result.

A main advantage of the one-level model is its down-compatibility to traditional feedforward and feedback queue scheduling alternatives. In essence straight forward estimations of the multirate ensemble statistic create the desired parameters of multirate scheduling schemes too.

The incomplete Gamma function proves to be a keying operational component of the closed or near explicit solutions for any multi-rate traffic engineering.