A Fast Prediction of the Coverage Area in UMTS Networks

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Abstract—

In this paper we present a method for a fast prediction of the coverage area on the uplink of a UMTS network with non-homogeneous traffic. The coverage radius of a base station depends on the interference distribution at this base station since it determines the transmit powers required from the MS. The interference comprises both the own cell interference and the other cell interference which again depend on each other. Our proposed model for the computation of the interference distribution is based on an iterative solution of fixed-point equations which describe the interdependence of the interferences at neighboring base stations. Furthermore, we develop an efficient algorithm based on Lognormal approximations to compute the mean and standard deviation of the other cell interference. We will show that our model is accurate and fast enough to predict efficiently the coverage area which is one of the key tasks in the planning process of UMTS networks.

I. Introduction

With the forthcoming introduction of the *Universal Mobile Telecommunication System* (UMTS) in Europe a sophisticated planning of the network is required. The use of *Wideband Code Division Multiple Access* (WCDMA), however, requires also new paradigms in wireless network planning. While in GSM capacity is a fixed term, it is influenced in WCDMA by the interference caused by all mobile stations (MS) on the uplink, as well as the transmit powers of the base stations (BS) or NodeB on the downlink. Due to the power control mechanisms in both link directions, the signals are transmitted with such powers that they are received with nearly equal strengths. Therefore, the distribution of the user locations must be taken into account in order to perform a thorough network planning including both the uplink and the downlink. While the downlink primarily limits the capacity of the system, the uplink determines the coverage of the network, see [1]. The prediction of the coverage requires the knowledge of interference distributions at all BS to compute the outage probabilities for every point in the considered area.

A detailed examination of the interference on the uplink, however, is not a very straightforward task. Due to the universal frequency reuse in UMTS, all users both in the considered cell and in the neighboring cells will contribute to the total interference, thus influencing the link quality in terms of received bit-energy-to-noise ratio (E_b/N_0). Apart from the previously mentioned direct influence, there is also an indirect effect in the system. Since an increase in interference results in a higher required transmission power of the MS, there is a feedback behavior on the other cells as well. It is obvious that in order to model interference adequately it is necessary to capture this feedback behavior by performing an iterative computation.

Most studies on interference found in the literature do not fully take these interactions between cells into account. Among the first papers in this field, [2] and later [3] introduced a relative other cell interference factor f as the ratio between other cell interference to the interference due to users in the same cell. A closed form expression of the f-factor can be found in [4], when both BS and MS are assumed to be distributed according to a spatial Poisson process. Similar simple approximations with a fixed interference factor can be found in [1] and [5]. A more sophisticated model is given in [6] and later extended in [7]. Contrary to the prior studies, these models derive distributions for other cell interference which are used to calculate capacity bounds.

In this paper we present an analytical model for the computation of the other cell interference. Based on a spatial traffic distribution and given BS positions we use fixed-point equations to determine the distribution of the other cell interference. This iterative approach allows us to include the interdependence between the interferences of neighboring cells in our model which is not fully considered in previous work. With the knowledge of the other cell interference we are able to determine the own cell and finally the total interference. With the distribution of the total interference the coverage area for the different services is calculated.

The paper is organized as follows. Section II describes the system model and gives a basic idea about the iterative approach. In Section III the other cell interference is calculated for a given set of MS with fixed positions. This computation is extended in Section IV to a stochastic spatial user distribution. Furthermore, we will use Lognormal approximations to derive an algorithm for the efficient computation of the other cell interference distribution. A plot of the coverage area is shown in Section V by means of an example network with non-homogeneous traffic distribution. The paper is concluded in Section VI with a short outlook on future work.

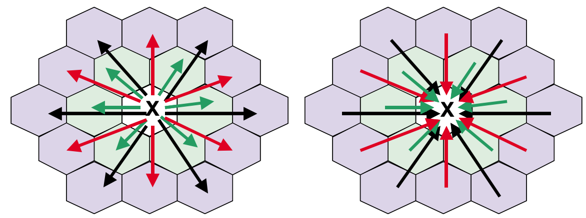
II. System Model and Basic Approach

The focus of this paper is to derive a method to determine the coverage area of large UMTS networks. The crucial point in doing so is the computation of the other cell interferences. Once the other cell interference is known the calculation of the total interference distribution and the resulting coverage area is a straightforward task. The network is defined by a set of BS in a hexagonal grid, a spatial user distribution, and a pathloss model. The example in Section V uses a spatial Poisson process [8] with different traffic densities in the cells. The signal attenuation between BS x and MS k is defined according to [9] as

$$d_{k,x} = -128.1 - 37.6\log_{10}(dist_{k,x}), \qquad (1)$$

with $dist_{k,x}$ being the distance between k and x in km and $d_{k,x}$ the signal attenuation in dB. Note that in the following linear values are written as $\hat{\alpha}$ while α denotes the value in decibels.

With these input parameters we derive the other cell interference distribution at all BS using an iterative approach. The basic problem and also its solution are illustrated in Fig. 1. The MS of the central BS x which are all MS that have their strongest signal at BS x - soft handover is neglected - cause the interference $\hat{I}_{x,y}$ at other BS y, see Fig. 1(a). And, depending on this interference, the MS of the other BS again induce interference $\hat{I}_{y,x}$ at the central BS. We formulate the relation between these interferences as fixed-point equations which are solved by repeated substitutions. In the first step the transmit power and thus the power received at other BS is computed without considering other cell interference. In the next iteration step we know the other cell interference



(a) Interference caused by the MS of BS x

(b) Interference at BS x by MS of other BS

Fig. 1. Illustration of the iterative model

from the previous step and include it in the calculation of the transmit powers and determine a new value for the other cell interference. The iteration finally converges since the other cell interference increases less in every step. In Section III the iteration is described for the deterministic case and in Section IV the stochastic model follows.

III. Deterministic Interference

In a WCDMA system the relation between the power $\hat{S}^R_{t,x}$ of a MS with service t received at its BS x and the interference $\hat{I}^{own}_x$ at this BS is given by the power control equation:

$$\hat{\varepsilon}^*_t = \frac{\frac{\hat{S}^R_{t,x}}{R_t}}{\hat{N}_0 + \hat{I}^{other}_x + \hat{I}^{own}_x - \frac{\hat{S}^R_{t,x}\nu_t}{W}} \qquad (2)$$

The other cell interference $\hat{I}^{other}_x$ corresponds to the sum of the powers received by the MS not power-controlled by BS x. Furthermore, $\hat{\varepsilon}^*_t$ denotes the target E_b/N_0, R_t the bitrate, and ν_t the activity factor of MS with service t. The thermal noise density is denoted by $\hat{N}_0$. Solving this equation for the received power yields

$$\hat{S}^R_{t,x} = W\frac{\hat{\varepsilon}^*_t R_t}{W + \hat{\varepsilon}^*_t R_t \nu_t}\left(\hat{N}_0 + \hat{I}^{other}_x + \hat{I}^{own}_x\right). \qquad (3)$$

In the following α_t is an abbreviation for the term $\hat{\varepsilon}^*_t R_t \left(W + \hat{\varepsilon}^*_t R_t \nu_t\right)^{-1}$, $\bar{\alpha}$ is the vector $(\alpha_1, ..., \alpha_T)$ where T is the number of available services. Further, $n_{t,x}$ denotes the number of MS belonging to BS and $\bar{n}_{\nu,x}$ stands for the vector $(n_{1,x}\nu_1, ..., n_{T,x}\nu_T)$. Then, the own cell interference is given by

$$\hat{I}_x^{own} = \frac{1}{W}\sum_{t=1}^{T} n_{t,x}\hat{S}_{t,x}^{R}\nu_t = \bar{n}_{\nu,x}\bar{\alpha}^T\left(\hat{N}_0 + \hat{I}_x^{other} + \hat{I}_x^{own}\right) \quad (4)$$

and solving for $\hat{I}_x^{own}$ yields

$$\hat{I}_x^{own} = \frac{\bar{n}_{\nu,x}\bar{\alpha}^T}{1-\bar{n}_{\nu,x}\bar{\alpha}^T}\left(\hat{N}_0 + \hat{I}_x^{other}\right) \quad (5)$$

Hence, the power received from a MS with service t is

$$\hat{S}_{t,x}^{R} = W\alpha_t\left(\hat{N}_0 + \hat{I}_x^{other} + \hat{I}_x^{own}\right) = W\frac{\alpha_t}{1-\bar{n}_{\nu,x}\bar{\alpha}^T}\left(\hat{N}_0 + \hat{I}_x^{other}\right) \quad (6)$$

For the calculation of the other cell interference $\hat{I}_x^{other}$ at BS x we are first interested in the interference $\hat{I}_{y,x}$ caused by the MS power-controlled by BS y. The power received at BS x by such a MS k is

$$\hat{S}_{k,x}^{R} = \hat{S}_{k,y}^{R}\frac{\hat{d}_{k,x}}{\hat{d}_{k,y}}. \quad (7)$$

The sum of these powers divided by the total bandwidth yields the interference density at BS x caused by all MS of BS y

$$\hat{I}_{y,x} = \sum_{k:BS(k)=y}\frac{\nu_k\hat{S}_{k,x}^{R}}{W} = \left(\hat{N}_0 + \hat{I}_y^{other}\right)F_{y,x}, \quad (8)$$

where $F_{y,x}$ stands for

$$F_{y,x} = \sum_{k:BS(k)=y}\frac{\nu_k\alpha_k}{1-\bar{n}_{\nu,x}\bar{\alpha}^T}\frac{\hat{d}_{k,x}}{\hat{d}_{k,y}}. \quad (9)$$

Hence, the other cell interference $\hat{I}_x^{other}$ and the total interference $\hat{I}_x$ at BS x are given as

$$\hat{I}_x^{other} = \sum_{y\neq x}\hat{I}_{y,x} \quad \text{and} \quad \hat{I}_x = \hat{I}_x^{other} + \hat{I}_x^{own}. \quad (10)$$

Obviously, the interference $\hat{I}_{x,y}$ produced by all MS of BS x at another BS y depends on the other cell interference at BS x and vice versa. However, the term $F_{x,y}$ is independent of the other cell interference and this can be used to iteratively calculate the other cell interference.

IV. Stochastic Interference Model

The model presented in the previous section was deterministic for a set of MS with fixed positions and a set of BS. Furthermore, the attenuations between MS and BS were known. In this section we extend this model to the stochastic case where both the number of the MS and their location with corresponding attenuations are random. The idea behind the analysis is to compute the distribution of the other cell interference by solving a set of fixed-point equations which describe the relations between the other cell interferences:

$$\mathcal{I}_{x,y} = \left(\hat{N}_0 + \mathcal{I}_x^{other}\right)\mathcal{F}_{x,y} \quad \text{and} \quad (11)$$

$$\mathcal{I}_x^{other} = \sum_{y\in\mathcal{N}(x)}\mathcal{I}_{y,x}, \quad (12)$$

where $\mathcal{N}(x)$ denotes the set of BS which are near to x. In our case the BS of the first three tiers around BS x are considered, see Fig. 2. The r.v. $\mathcal{F}_{x,y}$ is the stochastic rep-

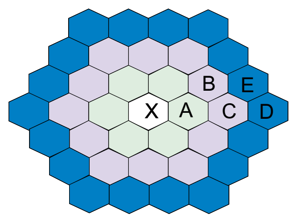


Fig. 2. Neighborhood of x

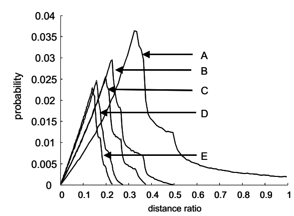


Fig. 3. Distance ratio PDF

resentation of the variable $F_{x,y}$ in Eq. (8) which remains constant throughout the iteration. This r.v. describes all the stochastic influences of the underlying spatial distribution. According to the theorem of total probability $\mathcal{F}_{x,y}$ is given as

$$\mathcal{F}_{x,y} = \sum_{\bar{n}:\bar{n}\bar{\alpha}^T<1} p(\bar{n},x)\frac{1}{1-\bar{n}\bar{\alpha}^T}\sum_{t=1}^{T}\alpha_t\sum_{i=1}^{n_{t,x}}\mathcal{D}_{x,y}, \quad (13)$$

where $p(\bar{n},x)$ is the probability that $n_{t,x}$ MS with service t are active in BS x. If in mean N_x users belong to BS x and q_t is the probability for service t the probability $p(\bar{n},x)$ is given according to the product form solution, see e.g. [10]

$$p(\bar{n},x) = \frac{\prod_{t=1}^{T}\frac{(N_x q_t \nu_t)^{n_t}}{n_t!}}{\sum_{\bar{n}':\bar{n}'\bar{\alpha}^T<1}\prod_{t=1}^{T}\frac{(N_x q_t \nu_t)^{n_t'}}{n_t'!}}. \quad (14)$$

If $\bar{n}\bar{\alpha}^T$ exceeds 1 the system's pole capacity is reached and the A_{out}-case according to [5] occurs. The r.v. $\mathcal{D}_{x,y}$

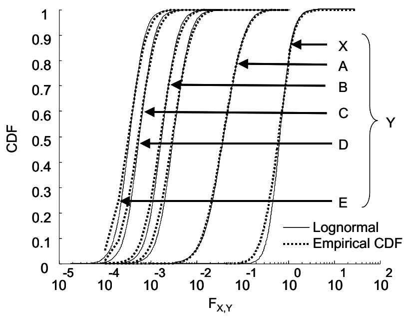


Fig. 4. Verification of the Lognormal distributions of $\mathcal{F}_{x,y}$

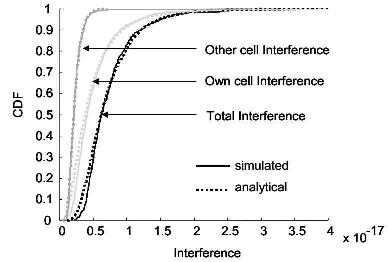


Fig. 5. CDF of other cell, own cell, and total interference

describes the spatial component of the arrival process. It is a r.v. for the ratio between the attenuations $\hat{d}_{k,y}$ and $\hat{d}_{k,x}$ while the MS k is located randomly within the cell of BS x. In case of the pathloss model given in Eq. (1) the distribution of the attenuation ratio corresponds to the ratio of the distance ratio power 3.76. The distance ratios for the five different pairs of BS occurring with three tiers are given in Fig. 3. Note that for $\mathcal{F}_{x,x}$ Eq. (11) yields the own cell interference at BS x. The exact computation of the distribution of $\mathcal{F}_{x,y}$ is numerically intractable, however, the r.v. approximately follows a Lognormal distribution which is shown in Fig. 4 for the different r.v. $\mathcal{F}_{x,y}$. Thus, the mean and variance of $\mathcal{F}_{x,y}$ are sufficient to describe the r.v. The calculation of the moments of $\mathcal{F}_{x,y}$ is again performed according to the theorem of total probabilities:

$$E[\mathcal{F}_{x,y}{}^{z}] = \sum_{\bar{n}:\bar{n}\bar{\alpha}^T<1} p(\bar{n},x) E[\mathcal{F}_{x,y}(\bar{n})^z] \quad (15)$$

The mean of $\mathcal{F}_{x,y}(\bar{n})$ is given directly by

$$E[\mathcal{F}_{x,y}(\bar{n})] = \frac{\bar{n}\bar{\alpha}^T}{1-\bar{n}\bar{\alpha}^T} E\left[\mathcal{D}_{x,y}\right] \quad (16)$$

while the second moment is calculated over the variance

$$E\left[\mathcal{F}_{x,y}(\bar{n})^2\right] = VAR\left[\mathcal{F}_{x,y}(\bar{n})\right] + E\left[\mathcal{F}_{x,y}(\bar{n})\right]^2. \quad (17)$$

The variance is given as

$$VAR\left[\mathcal{F}_{x,y}(\bar{n})\right] = \sum_{t=1}^{T} \frac{n_t\alpha_t^2}{(1-\bar{n}\bar{\alpha}^T)^2} VAR\left[\mathcal{D}_{x,y}\right]. \quad (18)$$

Finally, the variance of $\mathcal{F}_{x,y}$ is calculated as

$$VAR\left[\mathcal{F}_{x,y}\right] = E\left[\mathcal{F}_{x,y}^2\right] - E\left[\mathcal{F}_{x,y}\right]^2. \quad (19)$$

Note, that the sum over all user combinations $\bar{n}$ is performed only once since in each summation step the first and the second moment of all r.v. $\mathcal{F}_{x,y}(\bar{n})$ are calculated.

With the mean and variance of $\mathcal{F}_{x,y}$ known for all BS x and y we can solve the fixed-point equations given in Eq. (11) and (12). Starting without other cell interference the equations are solved by a repeated substitution of $\mathcal{I}_x^{other}$ into Eq. (11) and vice versa. The whole computation is performed under the assumption that the $\mathcal{F}_{x,y}$ follow a Lognormal distribution and that all involved r.v. are independent. Thus, the multiplication of Eq. (11) is done by adding the parameters μ and σ^2 of the Lognormal distributed r.v. $\mathcal{F}_{x,y}$ and $\hat{N}_0 + \mathcal{I}_x^{other}$. The distribution of $\mathcal{I}_x^{other}$ is also Lognormal since starting with $\mathcal{I}_x^{other} = 0$ we receive the $\mathcal{I}_{x,y}^{out}$ to be Lognormal. According to [11] the sums of slightly different Lognormal distributions are most likely Lognormal, as well, and hence is $\mathcal{I}_x^{other}$ as the sum of the $\mathcal{I}_{x,y}^{out}$ which follow Lognormal distributions. These distributions are added by summing the means and variances since independence is assumed. The iteration converges if the relative change of the mean and the variance of all other cell interferences fall below a certain threshold. Since the resulting other cell interference and the r.v. $F_{x,x}$ are Lognormal we can compute the own cell interference analogous. The total interference is finally derived by adding the mean and variance of own and other cell interference. In Fig. 5 the CDF of the other cell, the own cell, and the total interference are shown. They are compared with simulated values. They are obtained by generating snapshots of the spatial traffic distribution and computing interferences for each of these. The method is described in [12] in more detail.

V. Numerical Results

The main topic of this paper is to predict coverage probabilities of a UMTS network with multiple BS that offer different services. The p-percent coverage area is defined as the area in which the outage probability is smaller than (1-p) percent. The outage probability for a MS k with a maximum power $\hat{S}_{k,max}$ and attenuation $\hat{d}_{k,x}$ to its BS x is the probability that the maximum possible received power $S^R_{k,max} = \hat{S}_{k,max}\hat{d}_{k,x}$ is smaller than the required received power for the MS service t

$$p_k^{out} = p\left\{\hat{S}^R_{k,max} < W\alpha_t\left(\hat{N}_0 + \hat{I}_x\right)\right\}. \qquad (20)$$

Thus, with the p-percent-quantile $\hat{I}_x(p)$ of $\mathcal{I}_x$ we can determine the maximum possible attenuation $\hat{d}_{t,max}$ for service t that does not exceed the maximum outage probability

$$\hat{d}_{max,t} = W\alpha_t(\hat{N}_0 + \hat{I}_x(p))\hat{S}^{-1}_{max}. \qquad (21)$$

Since we assume a model that defines the pathloss according to the distance the coverage area of a BS is within a certain radius. In Fig. 6 and Fig. 7 the 90% coverage area determined analytically and by simulation are shown. We have considered the following services:

t	1	2	3	4	5
q_t	0	0.75	0.2	0.05	0.01
R_t[kbps]	8	12.2	64	144	384
ε_t^*[dB]	6.5	5.5	4	3.5	3

The traffic densities for the different BS are directly given in the coverage plots.

VI. Conclusion and Outlook

In this paper we presented an analytical model for computing the other cell interference in a UMTS system with multiple services. Our approach is based on solving a fixed-point equation which describes the interdependence between the interferences at neighboring BS. An efficient algorithm is used to solve these equations using Lognormal approximations such that the model can be used in the planning process of UMTS networks to predict the pulink coverage areas. Our future work consists of an extension of our model to soft handover. Furthermore, we will include a model for the downlink as well.

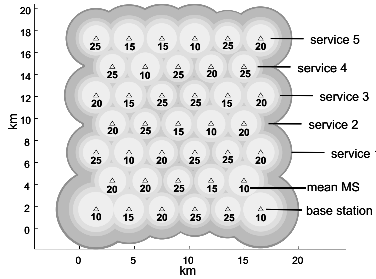


Fig. 6. Analytical result

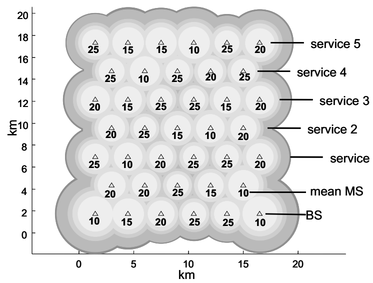


Fig. 7. Simulation

References

[1] H. Holma and A. Toskala, *WCDMA for UMTS*, John Wiley & Sons, Ltd., June 2000.

[2] A.M. Viterbi and A.J. Viterbi, "Erlang capacity of a power controlled CDMA system," *IEEE Journal on Sel. Areas in Comm.*, vol. 11, no. 6, pp. 892–893, August 1993.

[3] A.J. Viterbi, A.M. Viterbi, and E. Zehavi, "Other-cell interference in cellular power-controlled CDMA," *IEEE Trans. on Comm.*, vol. 42, no. 2/3/4, pp. 1501–1504, February/March/April 1994.

[4] P.J. Fleming, A.L. Stolyar, and B. Simon, "Closed-form expressions for other-cell interference in cellular CDMA," UCD/CCM 116, University of Colorado, Denver, CO, 1997.

[5] V. V. Veeravalli, A. Sendonaris, and N. Jain, "CDMA coverage, capacity and pole capacity," in *Proceedings of the 47th IEEE VTC*, Phoenix, AZ, May 1997, pp. 1450–1454.

[6] J. Evans and D. Everitt, "On the teletraffic capacity of CDMA cellular networks," *IEEE Trans. on Veh. Tech.*, vol. 48, no. 1, pp. 153–165, January 1999.

[7] G. Karmani and K.N. Sivarajan, "Capacity evaluation for CDMA cellular systems," in *Proc. of IEEE INFOCOM*, Anchorage, Alaska, April 2001.

[8] D. Stoyan and H. Stoyan, *Fractals, Random Shapes and Point Fields: Methods of Geometrical Statistics*, Wiley, 1994.

[9] ETSI, "Selection procedures for the choice of radio transmission technologies of the UMTS," Technical Report TR 101 112 V3.2.0, ETSI SMG-5, Sophia Antipolis, France, April 1998.

[10] J.S. Kaufman, "Blocking in a shared resource environment," *IEEE Transactions on communications*, vol. 29, no. 10, pp. 1474–1481, October 1981.

[11] E. Barouch, G.M. Kaufmann, and M.L. Glasser, "On sums of lognormal random variables," *Studies in Applied Math.*, vol. 75, pp. 37–55, 1986.

[12] D. Staehle, K. Leibnitz, K. Heck, B. Schröder, A. Weller, and P. Tran-Gia, "Analytical characterization of the soft handover gain in UMTS," in *Proc. of VTC'01 Fall*, Atlantic City, NJ, October 2001.