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Andreas Mäder and Dirk Staehle

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University of Würzburg,
Dept. of Computer Science,
Am Hubland, 97074 Würzburg, Germany
[maeder, staehle]@informatik.uni-wuerzburg.de

Analytic Modelling of the WCDMA Downlink Capacity in Multi-Service Environments

Andreas Mäder* and Dirk Staehle

Department of Distributed Systems, Institute of Computer Science,

University of Würzburg,

Am Hubland, 97074 Würzburg, Germany

Abstract: The *Universal Mobile Telecommunication System* (UMTS) operates with *Wideband Code Division Multiple Access* (WCDMA) over the air interface. In the literature, the CDMA uplink, e.g. as implemented by the IS-95 standard, receives most attention due to its limiting influence on the overall network capacity in voice only environments. But with the growing demand of the mobile user for multimedia content, the 3G systems like UMTS provide service classes specifically designed for asymmetric traffic such as video streams. This makes the WCDMA downlink (forward link) the potential bottleneck for the network capacity. Another distinction of WCDMA to IS-95 is the introduction of fast power control mechanisms on the downlink. In this paper, we propose an analytic algorithm to approximate the WCDMA downlink cell capacity in a multi-service environment. It is based on a recursion formula and includes the effects of *soft blocking* and *imperfect power control*. Due to its design, the algorithm is time and memory efficient and is well suited for the use in network planning tools for e.g. capacity planning or dimensioning issues.

Keywords: *UMTS, WCDMA, downlink, forward link, capacity, soft blocking*

1 Introduction

The *Wideband Code-Division Multiple Access* (WCDMA) scheme is proposed and implemented as air interface of the 3G and 3.5G mobile communication networks such as UMTS. The predecessors of this technology, such as IS-95 from Qualcomm, were primarily designed for voice-traffic. Since this kind of traffic is symmetric in respect of up- and downlink, the uplink is the limiting factor for the overall network capacity. This fact is reflected in the high amount of literature regarding the CDMA uplink. One of the first papers in this field

*Corresponding author:maeder@informatik.uni-wuerzburg.de, Tel.: ++49 931 8886642, Fax: ++49 931 8886632

was [1]. Others such as [2, 3, 4, 5, 6] included multiple services, imperfect power control, user activity, soft handover and othercell interference in their studies.

All these works concentrated on the CDMA and WCDMA uplink, respectively. In 3G systems, however, it is expected that asymmetric data transfers such as video streams or web traffic will play a major role in the overall traffic volume and the downlink thus is becoming the limiting factor of the cell capacity. So, in this scenario the downlink becomes a crucial factor for the proper dimensioning and planning of mobile networks. It is therefore important to find algorithms which are able to estimate the downlink capacity of WCDMA networks in a time and memory efficient way to ensure that the mobile network providers can use them in their planning tools.

In the literature, most approaches for evaluating the downlink performance rely on Monte-Carlo simulations, such as in [7] or [8]. The authors of the latter use sophisticated simulation techniques to reduce the required simulation time for the computation of coverage and service availability probabilities. One of the earlier works using analytical methods is [9]. The authors introduce a closed form for the outage probability in the case of no multipaths, and a Chernoff bound in case of several multipaths. The capacity is calculated by setting a boundary for the outage probabilities which has to be satisfied. In [10], the capacity is calculated for voice and data users with additional consideration of signaling and shared channels. A recursive scheme for reducing the computational complexity is used.

One of the most prominent properties of WCDMA systems is the so called *soft capacity*. This term denotes the fact that in a WCDMA system, capacity is not a deterministic but a stochastic value. This results from the various effects of multipath propagation, pseudo-orthogonality on the uplink, thermal noise, etc., on the received interference at the mobiles and NodeB, respectively. Hence, the term *soft blocking* and resulting from that, *soft capacity*, describes the possibility that virtually in every system state blocking of an incoming connection is possible, but with varying probabilities. In our work we propose an analytic algorithm for the calculation of blocking probabilities which includes the effects of soft blocking. The proposed algorithm is time and memory efficient making it well suited for the use in planning tools.

The paper is organized as follows: In Section 2, a more detailed description of the problems arising from the capacity determination in WCDMA is given. In Section 3, we describe our system model including the interference model and develop a model for the calculation of the transmission power depending on the number of power controlled mobiles. Based on this model, in Section 4 an algorithm for the calculation of downlink blocking probabilities and downlink capacity of a WCDMA cell is introduced. In addition, the approximation algorithm is proposed, which is validated and used in a short parameter study shown in Section 5.

2 Problem Formulation

In contrast to the WCDMA uplink, where the received Multiple Access Interference (MAI) at the NodeB defines the cell capacity, the downlink is limited by the maximum transmission power. The NodeB has to satisfy the E_b/N_0-requirements of all power controlled mobiles. The required power level not only depends on the QoS-requirements of the service, but also on the position of the mobile in the cell[1]. Furthermore, the use of orthogonal spreading codes reduces the intracell interference at the MS, but in practice this effect is reduced by multipath propagation. Therefore, an orthogonality factor $\alpha \leq 1$ is introduced describing the fraction of power which is seen as interference by mobiles power controlled by the same NodeB.

In WCDMA as implemented by UMTS, fast power control is responsible for meeting the minimum required transmission powers to each mobile on the downlink. In mathematical terms, these minimum powers are defined by the power control equation as shown in (1):

$$\hat{\varepsilon}^*_{k,x} = \frac{W}{R_k} \frac{\hat{S}_{k,x}\hat{d}_{k,x}}{W\hat{N}_0 + \sum_{y \neq x} \hat{S}_y \hat{d}_{k,y} + \alpha \hat{d}_{k,x}(\hat{S}_x - \hat{S}_{k,x})}, \tag{1}$$

where

[1] A cell in this context is equivalent to a sector

- $\hat{\varepsilon}^*_{k,x}$ is the target-E_b/N_0-value for mobile k at NodeB x,
- $\hat{S}_{k,x}$ is the transmit power for MS k from NodeB x,
- $\hat{d}_{k,x}$ is the corresponding attenuation factor,
- $\hat{N}_0$ is the thermal noise,
- W is the system bandwith in Mcps,
- R_k is the requested bitrate
- and $\hat{S}_x$ is the total transmission power of NodeB x.

The power control equation defines the theoretical minimum transmission powers. With the assumption that the power control works optimal, we speak of *perfect power control*. In reality, however, the desired target-E_b/N_0 values are not exactly achieved. Instead, the received powers fluctuate slightly around the optimal value. This behaviour is called *imperfect power control*. Measurements in CDMA networks [2] have shown that the received target-E_b/N_0 values are normal distributed in the dB-domain, hence can be modelled with a lognormal distribution.

Our goal is to develop an analytic algorithm to calculate the blocking probabilities of each service class s in a cell belonging to NodeB x. Then, with reasonably chosen service dependent boundaries for the blocking probabilities, the overall capacity of a cell can be calculated.

The call admission control (CAC) in WCDMA on the downlink estimates the increase of transmission power that an incoming connection, i.e. radio bearer, causes. This increase depends on the service class of the radio bearer, since the different service classes have different QoS-requirements which are reflected in the transmission power $S_{k,x}$. If the newly estimated transmission power is above a certain threshold, the call will be blocked. So, in order to calculate blocking probabilities it is required to know the total transmission power of a NodeB depending on the number of power controlled mobiles. This is done by using a similar approach as in [11].

3 System Model

We consider a UMTS network with $\mathbf{L}$ NodeBs from which x is the examined NodeB and $\mathbf{Y} = \mathbf{L} \setminus \{x\}$ is the set ff surrounding NodeBs. The set $\mathbf{K}_x$ comprises the mobile stations which are power controlled by the examined NodeB x, from which $\mathbf{A}_x$ mobiles are active. The coverage area $\mathbf{F}_x$ of the NodeB x is partitioned in subareas f with a Poisson distributed number of users offering a traffic density of a_f. The probability that a specific mobile requests a radio bearer of service class s is p_s. A service class is defined by its bitrate R_s and its required target-E_b/N_0-value $\hat{e}_s^*$.[2] The set of service classes is $\mathbf{S}$. We consider imperfect power control, so the experienced E_b/N_0-value is a normal distributed random variable in the dB-domain and a logormal random variable in the linear domain. The distances of the NodeBs to the subareas f lead to specific attenuation factors $d_{k,x}$. We use the determistic model from [12]:

$$d_{k,x} = -128.1 - 37.6\log_{10}(\text{dist}(x,k)) \tag{2}$$

where $\text{dist}(x,k)$ is the distance between NodeB x and an mobile k in area f. Throughout the paper, we assume a thermal noise density of $N_0 = -174\text{dBm/Hz}$ and a system chiprate of 3.84Mcps. The transmission powers of the NodeBs $y \neq x$ are modelled as independent lognormal random variables including an constant power fraction $\hat{S}_c$ reflecting the common channels. One cell is modelled with $|\mathbf{S}|$ Poisson arrival processes and exponentially distributed service times with load a_s per service class s.

3.1 Interference Model

Since the call admission control (CAC) on the WCDMA downlink is based on the maximum transmission power of a NodeB, it is crucial to know the transmit power a MS k receives

[2] Note that we denote a linear value by $\hat{a}$ while a is in decibels

from a NodeB x. From Eq. (1) the transmit power $\hat{S}_{k,x}$ is derived as

$$\hat{S}_{k,x} = \omega_k \left(W\hat{N}_0\hat{\delta}_{x,k} + \sum_{y\in\mathbf{Y}} \hat{S}_y\hat{\Delta}_{y,k} + \alpha\hat{S}_x \right). \tag{3}$$

The term ω_k is the *service load factor* defined similar as in [5], but with consideration of the orthogonality factor:

$$\omega_k = \frac{\hat{\varepsilon}_k R_k}{W + \alpha\hat{\varepsilon}_k R_k} \qquad \text{and} \qquad \omega_{k,y} = \begin{cases} \omega_k\hat{\delta}_{k,x} & \text{if } y = 0 \\ \omega_k\alpha & \text{if } y = x \\ \omega_k\hat{\Delta}_{k,y} & \text{if } y \neq x \end{cases} \tag{4}$$

Consequently, the sum of the service load factors of all active mobiles power controlled by NodeB x defines the load η_x of NodeB x:

$$\eta_x = \sum_{k\in\mathbf{A}_x} \omega_k \qquad \text{and} \qquad \eta_{x,y} = \sum_{k\in\mathbf{A}_x} \omega_{k,y} \tag{5}$$

The random variable $\hat{\delta}_{k,x}$ is the reciprocal of the attenuation factor $\hat{d}_{k,x}$, and $\hat{\Delta}_{k,y}$ is defined as the ratio of the attenuations from NodeB y to MS k and NodeB x to MS k: $\hat{\Delta}_{k,y} = \frac{\hat{d}_{k,y}}{\hat{d}_{k,x}}$. Now, if we sum over all transmit powers $S_{k,x}$ we get the total transmission power of NodeB x:

$$\hat{S}_x = \eta_{x,0}W\hat{N}_0 + \sum_{y\in\mathbf{Y}} \eta_{x,y}\hat{S}_y + \eta_{x,x}\hat{S}_x + \hat{S}_c \tag{6}$$

$$\Leftrightarrow \quad \hat{S}_x = \frac{1}{1-\eta_{x,x}} \left(\eta_{x,0}W\hat{N}_0 + \sum_{y\in\mathbf{Y}} \eta_{x,y}\hat{S}_y + \hat{S}_c \right) \tag{7}$$

where $\hat{S}_c$ is the contribution of the signaling and common shared channels to the transmission power, which is assumed to be constant. Knowing the total transmission power we can now use it within the admission control condition. A mobile is granted a new radio bearer if it holds that

$$
\begin{aligned}
\hat{S}_x < \hat{S}_{\max} \quad &\Leftrightarrow \quad \frac{1}{1-\eta_{x,x}}\left(\eta_{x,0} W \hat{N}_0 + \sum_{y\in\mathbf{Y}} \eta_{x,y}\hat{S}_y + \hat{S}_c\right) < \hat{S}_{\max} \\
&\Leftrightarrow \quad \eta_{x,0} W \hat{N}_0 + \sum_{y\in\mathbf{Y}} \eta_{x,y}\hat{S}_y + \hat{S}_c < \hat{S}_{\max} - \eta_{x,x}\hat{S}_{\max} \\
&\Leftrightarrow \quad \eta_{x,0} W \hat{N}_0 + \sum_{y\in\mathbf{Y}} \eta_{x,y}\hat{S}_y + \eta_{x,x}\hat{S}_{\max} < \hat{S}_{\max} - \hat{S}_c \\
&\Leftrightarrow \quad \sum_{k\in\mathbf{A}_x} \omega_k \left(W\hat{N}_0 \delta_{k,x} + \sum_{y\in\mathbf{Y}} \Delta_{k,y}\hat{S}_y + \alpha\hat{S}_{\max}\right) < \hat{S}_{\max} - \hat{S}_c.
\end{aligned}
\tag{8}
$$

Now, if we define the random variable Q_k as

$$
Q_k = W\hat{N}_0\delta_{k,x} + \sum_{y\in\mathbf{Y}} \Delta_{k,y}\hat{S}_y + \alpha\hat{S}_{\max}, \tag{9}
$$

the admission control condition (8) can be rewritten as

$$
\hat{S}_x^* < S_{\max} - \hat{S}_c \quad \text{with} \quad \hat{S}_x^* = \sum_{k\in\mathbf{K}_x} \nu_k \omega_k Q_k. \tag{10}
$$

The "dedicated" transmission power $\hat{S}_x^*$ now considers all mobiles $\mathbf{K}_x$ which are power controlled by NodeB x. The activity factor ν_k reflects the service dependent Bernoulli activity of each mobile. The variable Q_k depends on the position of the MS k only. For this reason we refer to it as *positional load factor* in the remaining of the paper.

One possibility is now to use an event-driven simulation to calculate the blocking probabilities depending on the number of mobile stations K. This requires a relatively long simulation time due to the large number of stochastic factors wich have to be considered. For this reason, we model $\hat{S}_x^*$ as lognormal distributed random variable following [13], and are then able to calculate the blocking probabilities for a specific number of mobiles analytically. This means that we have to know the first and second moment of the left hand side in the blocking condition in dependence on the current number of mobiles.

The first moment of $\hat{S}_x^*$ is derived easily from (3) and (9):

$$\begin{aligned} E[\hat{S}_x^*] &= E[\sum_{k\in\mathbf{K}_x} \nu_k \omega_k Q_k] \\ &= \sum_{k\in\mathbf{K}_x} \nu_k E[\omega_k] E[Q_k] \end{aligned} \tag{11}$$

and

$$E[Q_k] = W\hat{N}_0 E[\hat{\delta}_x] + \sum_{y\in\mathbf{Y}} E[\hat{\Delta}_y]E[S_y] + \alpha \hat{S}_{\max} \tag{12}$$

Note that we assume that the location of the mobiles are i.i.d. so it holds that $E[\hat{\delta}_{k,x}] = E[\hat{\delta}_x]$ and $E[\hat{\Delta}_{k,y}] = E[\hat{\Delta}_y]$. The moments of $\hat{\delta}_{k,x}$ and $\hat{\Delta}_{k,x}$ are obtained by summing over all subareas $f \in F_x$. The probability that a subarea is within the coverage area of NodeB x is given by

$$p(f \in F_x) = P(\hat{d}_{k,x} = \min\{\hat{d}_{k,y}\}), \quad x, y \in L \tag{13}$$

and the overall traffic intensity at NodeB x is

$$a_{x,s} = p_s \sum_{f\in\mathbf{F}_x} a_f p(f \in \mathbf{F}_x). \tag{14}$$

Then, the means of $\hat{\delta}_{k,x}$ and $\hat{\Delta}_y$ are given by

$$E[\hat{\delta}_x] = \sum_{f\in\mathbf{F}_x} \frac{a_f p(f\in\mathbf{F}_x)}{\sum_{s\in\mathbf{S}} a_{x,s}} E\left[\tfrac{1}{\hat{d}_{f,x}} | f \in \mathbf{F}_x\right] \tag{15}$$

and

$$E[\hat{\Delta}_y] = \sum_{f\in\mathbf{F}_x} \frac{a_f p(f\in\mathbf{F}_x)}{\sum_{s\in\mathbf{S}} a_{x,s}} E\left[\tfrac{\hat{d}_{f,y}}{\hat{d}_{f,x}} | f \in \mathbf{F}_x\right]. \tag{16}$$

The second moment of $\hat{S}_x^*$ is given by

$$\begin{aligned} E[\hat{S}_x^{*2}] &= \sum_{k\in\mathbf{K}_x} E[(\nu_k\omega_k Q_k)^2] \\ &= \sum_{\substack{k_1\in\mathbf{K}_x \\ k_1\neq k_2}} \sum_{k_2\in\mathbf{K}_x} \nu_{k_1}\nu_{k_2} E[\omega_{k_1}]E[\omega_{k_2}]E[Q_{k_1}Q_{k_2}] + \sum_{k\in\mathbf{K}_x} \nu_k^2 E[\omega_k^2]E[Q_k^2]. \end{aligned} \tag{17}$$

Furthermore, the second moment of Q_k which is required for this calculation is

$$\begin{aligned} E[Q_k^2] =&(W\hat{N}_0)^2 E[\hat{\delta}_x^2] + 2W\hat{N}_0 \sum_{y\in\mathbf{Y}} E[\hat{S}_y]E[\hat{\Delta}_y\hat{\delta}_x] + 2\alpha\hat{S}_{\max}W\hat{N}_0E[\hat{\delta}_x] \\ &+ (\alpha\hat{S}_{\max})^2 + \sum\sum_{y_1\neq y_2} E[\hat{S}_{y_1}]E[\hat{S}_{y_2}]E[\hat{\Delta}_{y_1}\hat{\Delta}_{y_2}] + \sum_{y\in\mathbf{Y}} E[\hat{S}_y]E[\hat{\Delta}_y^2] \end{aligned} \tag{18}$$

and the combined moment of $Q_{k_1}Q_{k_2}$ is given by

$$\begin{aligned} E[Q_{k_1}Q_{k_2}] =&(W\hat{N}_0)^2 E[\hat{\delta}_x]^2 + 2W\hat{N}_0 \sum_{y\in\mathbf{Y}} E[\hat{S}_y]E[\hat{\Delta}_y]E[\hat{\delta}_x] \\ &+ 2\alpha\hat{S}_{\max}W\hat{N}_0E[\hat{\delta}_x] + \sum_{y_1}\sum_{y_2} E[\hat{S}_{y_1}]E[\hat{S}_{y_2}]E[\hat{\Delta}_{y_1}]E[\hat{\Delta}_{y_2}] \end{aligned} \tag{19}$$

Now, we can compute the probability that the maximum transmission power of NodeB x is exceeded. We approximate S_x^* as lognormal distributed random variable and calculate the probability that $S_x^* > S_{\max} - S_c$. Mean and variance of the distribution are provided by $E[S_x^*]$ and $E[S_x^{*2}] - E[S_x^*]^2$. Since this probability strongly depends on the number of power controlled mobiles and the cell load η_x, resp., we write the blocking probability as $\beta(\eta_x)$:

$$\beta(\eta_x) = 1 - \mathrm{LN}_{\mu,\sigma}(S_{\max}^* - S_c) \tag{20}$$

The location and shape parameters μ, σ are defined by the first moment and variance of $\hat{S}_x^*$:

$$\mu = \ln(E[\hat{S}_x^*]) - \tfrac{1}{2}\sigma \tag{21}$$

and

$$\sigma = \sqrt{\ln(\rho)^2 + 1)}, \tag{22}$$

where ρ denotes the coefficient of variation of $\hat{S}_x^*$.

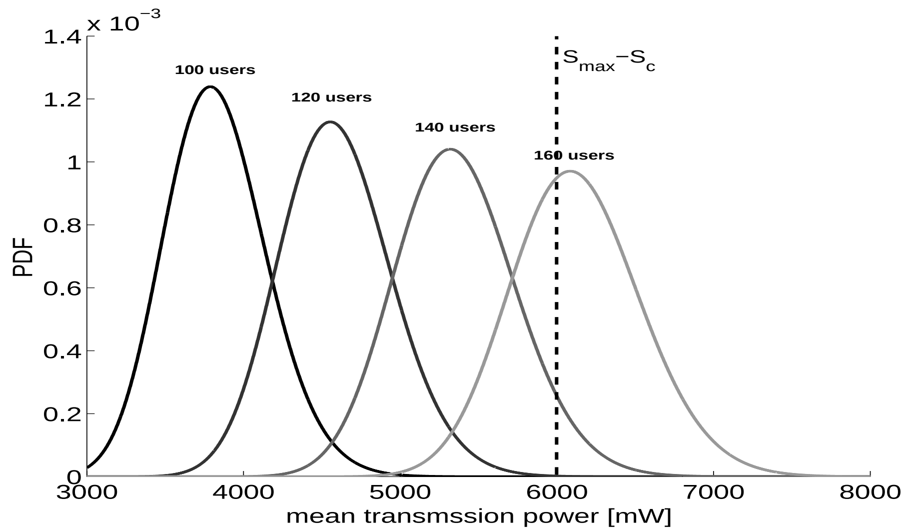

Figure 1: Local soft blocking probabilities depending on the mean transmission power

In Fig. 1, the pdfs of the dedicated transmission powers for different numbers of mobiles are shown for the 12.2kbps service with a maximum transmission power $S_{\max}$ of 8000mW and a constant part of 2000mW. The blocking probabilities correspond to the area between the right hand side of the line labelled $S_{\max} - S_c$ and the pdfs. The different curves exemplify different system states and show how the soft blocking probabilities are influenced by the cell load.

4 WCDMA Cell Capacity

The soft blocking probabilities we obtained in the previous section depend on the current system state, that is on the cell load. Therefore, these soft blocking probabilities can be seen as *local blocking probabilities.* However, for the calculation of the capacity of a WCDMA cell we are interested in the *total blocking probabilities* which depend on the offered load only. We obtain these probabilities by modifying the Markov arrival process to include the effects of soft blocking i.e. the local blocking probabilities.

In the previous section, the dedicated transmission power $\hat{S}_x^*$ already includes the incoming connection. But since the amount of radio resources the new connection requires depends on the service class, the resulting blocking probability β is service class specific too. We introduce the notation $\hat{S}_x^* + \bar{1}_s$ for the arrvial of a new connection j of service class s and define

$$E[\hat{S}_x^* + \bar{1}_s] = E[\hat{S}_x^*] + E[\omega_s]E[Q] \tag{23}$$

$$\begin{aligned} E[(\hat{S}_x^* + \bar{1}_s)^2] &= E[\hat{S}_x^{*2}] + 2E[\omega_s]E[QQ'] \sum_{t \in S} n_t \nu_t E[\omega_t] + E[\omega_s^2]E[Q^2] \\ &= E[\hat{S}_x^{*2}] + 2E[\omega_S]E[QQ']E[\eta_a] + E[\omega^2]E[Q^2] \end{aligned} \tag{24}$$

Note that we can assume here that $E[Q_k] = E[Q]$ and $E[\omega_k] = E[\omega_s]$ for connections k with service class s. $E[QQ']$ corresponds to the moment of the product of two positional load factors of different mobiles k and j, hence to $E[Q_k Q_j]$. The notation $E[\eta_a] = \sum_{s \in \mathbf{S}} n_s \nu_s E[\omega_s]$ is introduced for the sake of readability, with n_s as the number of connected mobiles of service class s. We further assume that the incoming connection is active, hence the activity factor is neglected. The service dependent soft blocking probabilities β_s are then calculated as in (20) but with the moments of $\hat{S}_x^* + \bar{1}_s$.

The local blocking probabilities are now applied to the transition rates of the S-dimensional Markov chain to reflect the effects of soft blocking. In Fig. 2, an example state space with the reduced transition rates for two service classes is shown.

The total blocking probabilities can be calculated as the sum of the total soft blocking probability and the hard blocking probability. The latter is simply the sum of the probabilities of the highest states for a specific service class. The total soft blocking probabilities are given by

$$P_{soft}(s) = \sum_{\bar{n} \in \Omega} \beta_s(\eta_x(\bar{n})) \bar{X}(\phi(\bar{n})). \tag{25}$$

The injective indexing function $\phi : \Omega \to \mathbb{N}$ maps the state space Ω to $\mathbb{N}$. The state

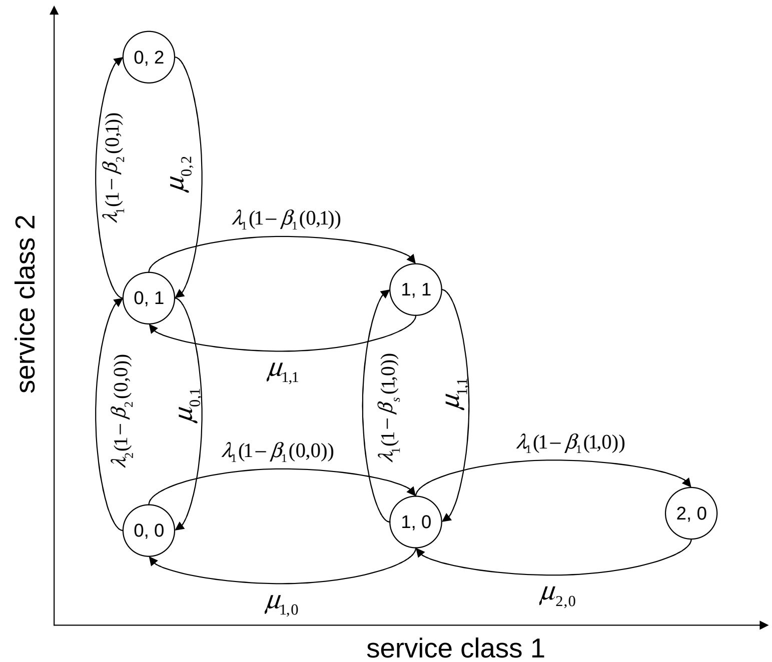

Figure 2: Example state space with two service classes

probability vector $\bar{X}$ is computed by solving the matrix equation

$$Q\bar{X}^T = 0, \tag{26}$$

where Q is the transition rate matrix.

4.1 Approximation Model of the State Space

The state space spanned by the system model is an S-dimensional space, which means that the number of states grows exponentially with the number of supported service classes. For this reason, we propose similar as in [6] a recursive approximation scheme based on [14]. States with similar resource occupations, that is with similar values of η_x, are combined to one bigger macro state. This folds the S-dimensional state space into on dimension, with transitions for the different resource requirements of the service classes.

In order to combine similar states, a common resource must be defined. In this case it is reasonable to choose the load factor η_x as resource, with the condition that $\eta_x < \eta_{\max}$. The maximal load $\eta_{\max}$ is implicitly given by the condition that it must hold that $\eta_x < \frac{1}{\alpha}$ for the feasibility of the power control equation. So, we define a basic resource unit g and map

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Note

resu

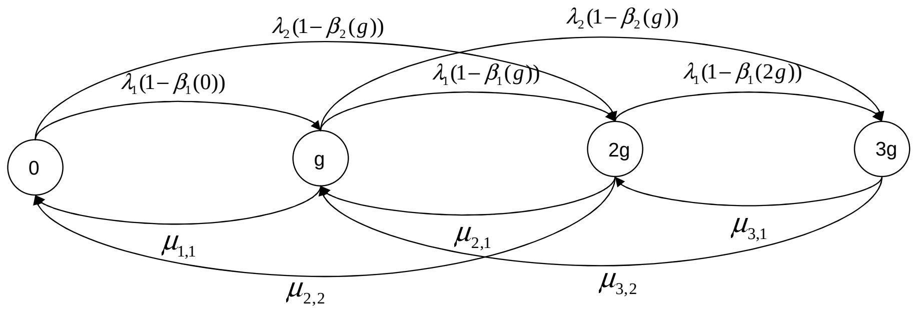

Figure 3: State diagram of the approximated state space

The recursion algorithm defined in [14] must be modified in order to include the local soft blocking probabilities. This leads to an approximation error since the recursion formula assumes that transitions in the same dimension have equal transition rates, which does not hold here because of the soft blocking. The modified recursion formula then becomes

$$\tilde{p}(\eta^*) = \frac{1}{\eta^*} \sum_{s \in \mathbf{S}} (1 - \beta_s(\eta^* - \psi_s))\tilde{p}(\eta^* - \psi_s) a_s \psi_s, \tag{28}$$

where η^* is the current system state and is a integer multiple of g. The state probability follows by normalizing $\tilde{p}$:

$$p(\eta^*) = \frac{\tilde{p}(\eta^*)}{\sum\limits_{jg \leq \eta_{\max}} \tilde{p}(jg)}, \quad j \in \mathbb{N}_0 \tag{29}$$

The calculation of the local blocking probabilities β_s requires the mean and variance of the dedicated transmission power depending on the current system load, hence $S_x^*(\eta^*)$. Since the actual number of connected mobiles is unknown, the moments of $S_x^*(\eta^*)$ must be computed recursively. This is done according to Eqs. (23) and (24). We set the transmission power to zero for $\eta^* = 0$ and obtain:

$$E[S_x^*(\eta^*)] = \begin{cases} 0 & \text{for } \eta^* = 0 \\ \sum\limits_{s \in \mathbf{S}} P_s(\eta^*) \left(E[S_x^*(\eta^* - \psi_s)] + \nu_s E[\omega_s] E[Q] \right) & \text{for } 0 < \eta^* \leq \eta_{\max} \end{cases} \tag{30}$$

and

$$E[S_x^*(\eta^*)^2] = \begin{cases} 0 & \text{for } \eta^* = 0 \\ \sum\limits_{s \in \mathbf{S}} P_s(\eta^*)(E[s_x^*(\eta^* - \psi_s)^2 + \nu_s E[\omega_s^2] E[Q^2] & \\ \qquad +2\nu_s E[\omega_s] E[QQ'] E[\eta_a(\eta^*)]) & \text{for } 0 < \eta^* \leq \eta_{\max} \end{cases} \tag{31}$$

A similar recursion scheme is used for the calculation of the mean cell load η_a:

$$E[\eta_a(\eta^*)] = \begin{cases} 0 & \text{for } \eta^* = 0 \\ \sum\limits_{s \in \mathbf{S}} P_s(\eta^*) \left(E[\eta_a(\eta^* - \psi_s) + \nu_s E[\omega_s] \right) & \text{for } 0 < \eta^* \leq \eta_{\max} \end{cases} \tag{32}$$

Note that in this case we include the activity factor ν_s since we are calculating the mean transmit power. For the soft blocking probabilities β_s, full activity of the incoming connection is assumed, so the activity factor is neglected. The rest of the calculation remains unmodified.

The probability $P_s(\eta^*)$ denotes the conditional probability that the current system state η^* has been reached from the predecessing state $\eta^* - \psi_s$ by an incoming connection of service class s. This probability is given by

$$P_s(\eta^*) = \frac{(1 - \beta_s(\eta^*))\tilde{p}(\eta^*) a_s \psi_s}{\sum\limits_{t \in \mathbf{S}} (1 - \beta_t(\eta^*))\tilde{p}(\eta^*) a_t \psi_t}. \tag{33}$$

Finally, the total blocking probabilities can be calculated. As in the multidimensional case they consist of hard and soft blocking:

$$P_{\text{block}}(s) = \underbrace{\sum_{jg<\eta_{\max}-\psi_s} \beta_s(jg)p(jg)}_{\text{soft blocking}} + \underbrace{\sum_{\eta_{\max}-\psi_s<kg\leq\eta_{\max}} p(kg)}_{\text{hard blocking}}, \quad j,k \in \mathbb{N}_0 \tag{34}$$

The soft blocking part reflects the possibility that in every system state η^* an incoming connection could be blocked due to the stochastic nature of the transmission power. The hard blocking part can be seen as blocking due to hardware restrictions if $\eta_{\max}$ is chosen properly, or, if no restrictions are given, due to the theoretical maximum of the cell capacity.

5 Numerical Results

In this section, we investigate the proposed algorithm in respect of accuracy and show its relative robustness against problematic scenarios. Furthermorse, we carry out a short parameter study in order to show the effects of several influencing variables on the performance of a WCDMA cell. The parameters which remain constant throughout the section if not stated otherwise are listed in Table 1.

system chiprate	3.84Mcps
number of surrounding NodeBs	6
cell layout	hexagonal
attenuation model	$d_{x,k} = -128.1 - 37.6\log_{10}(\text{dist}(x,k))$
distances between NodeBs	2km
mean transmission power of NodeBs in $\mathbf{Y}$	3000mW
standard deviation of transmission powers of NodeBs in $\mathbf{Y}$	200mW

Table 1: System parameters

The service definitions are shown in Table 2. The 12.2kbps service class corresponds to the voice service, whereas the classes with higher bitrates are designed for data services.

service class	1	2	3
bitrate	12.2kbps	64kbps	144kbps
target-E_b/N_0	5.5	4	3.5
E_b/N_0 standard deviation	1.2	1.2	1.2

Table 2: Service class definitions

The simulation we use for validating the analytical results is an event driven simulation designed to verify our assumptions. So it does not consider all features of UMTS but concentrates on the aspects critical for our analysis.

5.1 Analysis of the Algorithm

As stated in the previous sections, there exist several factors influencing the quality of the approximation the algorithm provides. The algorithm relies on the subsumption of states with similar load situations. This implies that the granularity of the basic resource unit g influences the approximation results, since with an increasing coarseness of the state space more states are subsumed into one macro state. Consequently, this leads to higher approximation errors. Another aspect is the approximation of the load distribution *within* an arbitrary system state with a lognormal distribution. This approximation is based on the assumption that in a state, the contribution of the different service classes to the load is similar. This assumption can be violated if in a scenario with several service classes the activity factors are very distinct from each other.

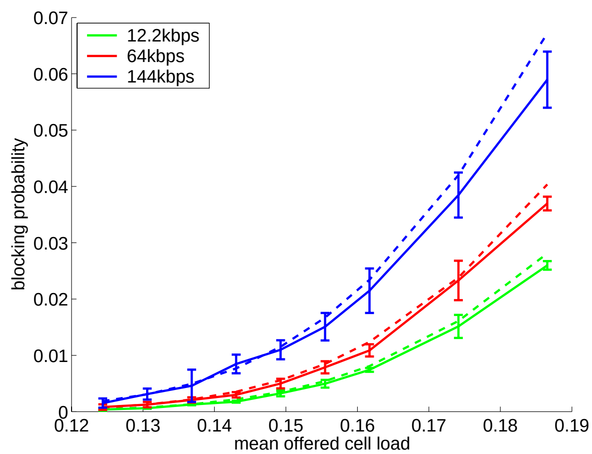

Figure 4: Scenario with three service classes and inhomogenous activity factors

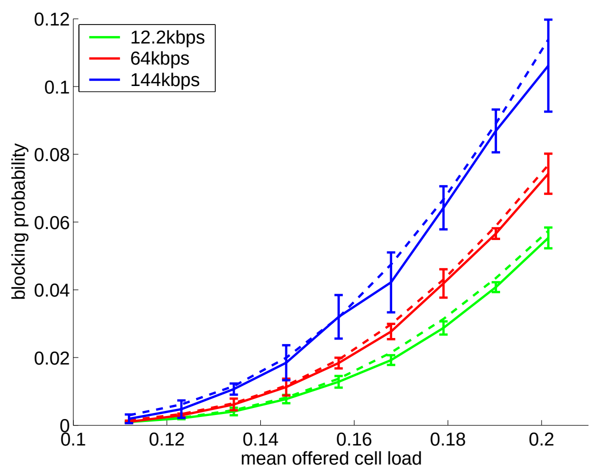

Figure 5: Scenario with always ON users

Such an scenario is illustrated in Fig. 4. The scenario consists of three service classes with bitrates of 12.2kbps, 66kbps and 144kbps. The activity factors ν_s are chosen very inhomogenously as 1, 0.6 and 0.1. The normalized mean offered cell load corresponding to η_x ranges from 0.10 to 0.16. These loads are assigned to the service classes with ratios of

0.70 for the first, 0.20 for the second and 0.05 of the third class. The inhomogenous activity factors lead to the overestimation of the blocking probabilities by the analytic algorithm (denoted by the dashed lines) in comparison to the simulated results (solid lines). Although there is a significant aberration in this scenario, the magnitude of the results are still quite good compared to the simulated results.

Fig. 5 shows blocking probabilities for the same service classes, but with always-ON users, i.e. with activity factors of 1. Since the contribution of the service classes to the load is now more homogenous, the approximation is significantly better than in the inhomogenous case. Especially the blocking probabilities for the 144kbps service class do not diverge to the extent as in the first scenario.

5.2 Parameter Study

Now we want to investigate the influence of several system parameters on the blocking probabilities and cell capacity, respectively. One interesting variable is the standard deviation of the target-E_b/N_0 values, which reflects the error induced by the imperfect power control. We define a scenario with three service classes as in the previous section but with more moderate activity factors of 0.4 for the voice class, 0.6 for the 64kbps service and 0.8 for the 144kbps service class. The service mix is again 0.70, 0.20 and 0.05. Furthermore, we define a set of maximum acceptable blocking probabilities as 1%, 3% and 5% for the three services.

We increase the standard deviation of the target-E_b/N_0 values from 0.2 to 1.6 and calculate the cell loads for which the acceptable blocking probabilities are not exceeded. The resulting curve is shown in Fig. 6. Since higher standard deviations increase the stochastic moment of the overall system, the cell capacity decreases. Note that typical values for the standard deviation are around 1.2 to 1.5, so the values left from the indicated area reflect the (unrealistic) case of a virtually perfect power control. It can also be stated that whithin this area, even small changes of the standard deviation leads to changing cell capacities.

Another interesting aspect is the impact of the transmission powers of the surrounding

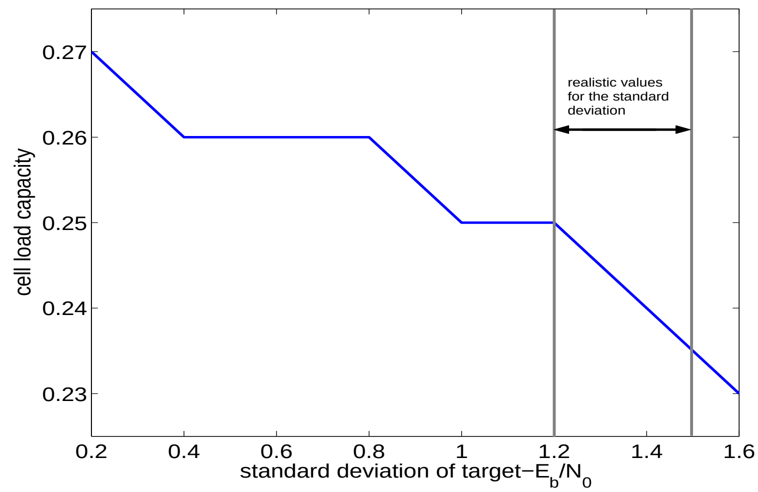

Figure 6: Influence of the target-E_b/N_0 standard deviation on the cell capacity

NodeBs (the othercell transmission power) on the cell capacity. It is obvious that a higher othercell transmission power leads to a lower cell capacity, so we focus on the effects of the variance of these power. For this reason, we increase the standard deviation of the transmission powers of all surrounding NodeBs from 0mW to 1000mW. The rest of the scenario is equal to the previous one.

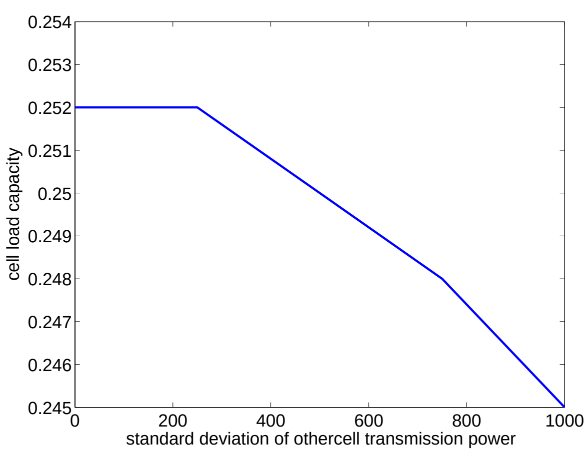

Figure 7: Cell capacity depending on the standard deviation of the othercell transmission power

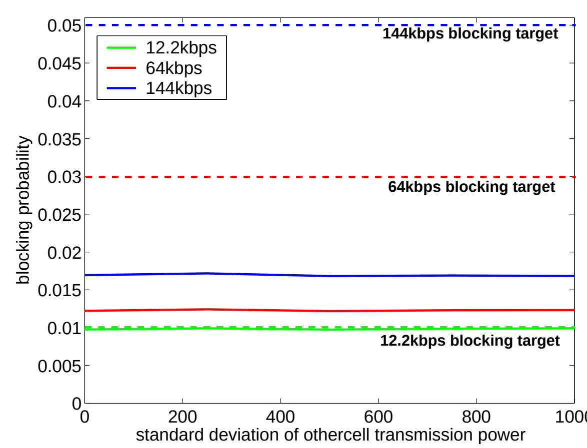

Figure 8: Corresponding blocking probabilities

The resulting curves are shown in Fig. 7 and 8. The first figure shows the correlation between standard deviation of the othercell tras nsmission power and the cell capacity. The cell capacity begins to decline at a deviation of ca. 200mW. These results show that the variance of the load of the surrounding cells must be taken into account for a proper network planning.

In Figure 8, the blocking probabilities corresponding to the cell capacities in Fig. 7 are illustrated. The dashed lines denote the target blocking probabilities of the service classes. In this scenario, the limiting service class is the 12.2kbps service, since its target blocking probability is the most stringent.

6 Conclusion

The goal of this paper was to develop an analytic algorithm suitable for the determination of WCDMA downlink cell capacities. We developed a system model which includes imperfect power control, i.e. the power control error, and introduced a service load factor describing the contribution of the various service classes to the cell load as well as a positional load factor wich depends on the position of a mobile in the cell. We modelled the call admission control in WCDMA with a blocking condition based on the current transmission power of the NodeB. The resulting soft blocking probabilities were used to modify the transition rates in an S-dimensional Markov chain to include soft capacity in the calculation of the overall blocking probabilities. In order to make the computation of the blocking probabilities and capacities feasible within an accepotable time margin, a recursive algorithm was introduced. The algorithm approximates the state space and provides results with a good match to the simulation. In the numerical results section we have shown that the power control error has a significant impact on the overall capacity of a WCDMA cell. Furthermore, we have seen the strong influence of the surrounding NodeBs' transmission powers on the cell capacity.

Several extensions of this work are in development. First, the transmission powers of the surounding NodeBs are calculated by our algorithm and are then used as inputs of an iterative algorithm which computes the capacity of a whole WCDMA network. Next, an algorithm which considers both up- and downlink with a two-dimensional state space reflecting up- and downlink loads is developed. Other aspects such as the soft handover gain will also be included.

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