An Analytic Method for Coverage Prediction in the UMTS Radio Network Planning Process

Dirk Staehle
Dept. of Distributed Systems
Institute of Computer Science
University of Würzburg, Germany
staehle@informatik.uni-wuerzburg.de

Abstract — One substantial element in the UMTS radio network planning process is the topology planning that identifies the NodeB sites and evaluates if the coverage and capacity requirements are met. The selection of the optimal sites is an iterative process of subsequently changing the network configuration and evaluating its quality. The latter task is commonly performed by quite time-consuming Monte Carlo simulations. In this paper we present an analytic method to determine the coverage area of a WCDMA network. This method yields slightly less accurate results in a considerably shorter time compared to a Monte Carlo simulation. Consequently, it should be used at the beginning of the planning process for a fast selection of cell sites. When a suitable solution is found, Monte Carlo or dynamic simulations should be used to validate the fine tuning of the site configuration. Although a lot of research focuses on the analytic modelling of WCDMA, few studies consider general WCDMA networks with arbitrary cell layout, spatial traffic distribution, service mix, and propagation losses. That however, is the prerequisite that makes an analytic model actually applicable in the planning process. Instead, most analytic methods focus on a single cell and approximate the impact from the other cells by the other-cell-to-own-cell interference ratio computed in [10] for hexagonal cells with homogeneous traffic distribution. This computation is refined in [11]. Both approaches for calculating the other-cell interference assume that the required received powers are equal at all cells and thus independent of the actual cell load. This assumption, however, does not even hold for equally loaded hexagonal cells. Instead, the required received power depends both on the load in the own-cell and the other cells. In [2] this effect is considered by subsequently computing the moments of the other-cell interference depending on the received power and vice versa until convergence is reached. However, the method still assumes a network with equally loaded cells and hence equally distributed received powers and other-cell interferences at different NodeBs. The approach is generalized in [8] for a network with arbitrary NodeB layout and heterogeneous spatial user distributions. However, it is restricted to the case of perfect power control and a deterministic propagation model without shadow fading. In this paper we remove both restrictions and additionally improve the time-efficiency of the algorithm. The rest of this paper is structured as follows. In Section II we formulate exactly how a WCDMA network and its coverage area are defined. We reduce the problem of computing the coverage area of the network to the problem of calculating the interferences in the network. Section III presents an algorithm for a time efficient approximation of the distribution of the interferences. Section IV shows the coverage area of an example network and how it depends on the system load and the applied shadow

I. INTRODUCTION

The planning process of WCDMA networks [3] consists of three planning phases: the pre-planning, the detailed planning, and the post-planning. The pre-planning phase produces a first estimation of the required site density and antenna height. The detailed planning phase actually selects the sites and configures them. The configuration includes e.g. the number of sectors and their orientation, the antenna parameters, and the pilot power. The post-planning phase takes place when the network is implemented in the operational status. Field measurements are made and key performance indicators are monitored to detect problems that require a re-planning of the network configuration. The general target of radio network planning and optimization is to find a network configuration with minimal costs that fulfills the service specific network requirements in terms of coverage, capacity, and QoS. One substantial element in all three planning phases is the topology planning that identifies the NodeB sites and evaluates if the coverage and capacity requirements are met. The selection of the optimal sites is an iterative process of subsequently changing the network configuration and evaluating the resulting quality. The latter task is commonly performed by quite
fading model. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

We define a WCDMA network configuration by the set \( B \) of NodeBs, the traffic map, the service mix, and the propagation model. The traffic map specifies the offered traffic (in Erlang) for every raster unit \( f \) and we assume that the number of users per area unit is a Poisson random variable with mean \( a f \). The service mix consists of a set \( S \) of \( S \) services and \( p_s \) is the probability for service \( s \). A service is further defined by its bit rate \( R_s \) and its target \( E_b/N_0 \) value \( \epsilon_s^* \). The imperfections of power control are taken into account by modelling the received \( E_b/N_0 \) value \( \epsilon_s \) by a normal random variable with mean \( \epsilon_s^* \) and standard deviation \( \sigma_s \). This leads to a load

\[
\omega_s = \frac{R_s \cdot \epsilon_s}{W + R_s \cdot \epsilon_s}
\]

as defined e.g. in [8]. The load \( \eta_x \) of NodeB \( x \) corresponds to the sum of the loads of all mobiles that are power-controlled by the NodeB. A mobile is power-controlled by the NodeB with maximum propagation gain and we define \( M_x \) as the set of mobiles belonging to NodeB \( x \). The power-controlling NodeB of a mobile \( k \) is denoted by \( B(k) \). Thus, we define the own-cell load of a NodeB as

\[
\eta_x = \sum_{k \in M_x} \omega_k
\]

Note that we write \( \omega_k \) for indicating the load of a specific mobile. The propagation model defines the path gain \( \gamma_{f,x} \) and the lognormal shadow fading \( \hat{\theta} \) between every area element \( f \) and NodeB \( x \). Accordingly, the propagation gain is \( d_{f,x} = \gamma_{f,x} \cdot \hat{\theta} \). The location of the NodeBs is inherently given through the path gains. Furthermore, the system chip rate is \( W = 3.84 \text{Mcps} \) and the thermal noise spectral density is \( N_0 = -174 \text{dBm} / \text{Hz} \). The multiple access interference at NodeB \( x \) is \( I_x \) and the other-cell interference is \( I_{x}^{\text{oth}} \).

In aim of this paper is to present an algorithm for deriving the coverage area of a WCDMA network consisting of several NodeBs. The \( p \)-percent coverage area corresponds to the area in which a user has an outage probability of less than \( p \) percent. According to [9] two kinds of outage probabilities exist. The first one, \( A_{\text{out}} \), covers the case that the pole capacity of a cell or network is exceeded and no solution for the power control equation system exists. The second one, \( B_{\text{out}} \), corresponds to the case that a solution for the power control equation system exists but the power demanded from a certain mobile exceeds the maximum transmit power of the mobile. The power required from a mobile depends on the propagation loss and consequently on the position of the mobile. The coverage area is defined by this second location dependent outage probability under the condition that the pole capacity of the system is not exceeded and the first kind of outage does not occur. In the following the term outage always refers
to a \( B_{\text{out}} \) event. The outage probability of a mobile with service \( s \) at location \( f \) depends on the interference at the surrounding NodeBs and its propagation loss to them. In fact, the interference and the service of a mobile define the required received power and the propagation loss relates the required received power to the required transmit power. Accordingly, the transmit power required from a mobile with service \( s \) at location \( f \) to maintain the desired signal quality at NodeB \( x \) is

\[
\hat{S}_{f,s,x}^* = \omega_s \cdot \frac{1}{\hat{\gamma}_{f,x} \cdot \hat{\theta}} \left( W \cdot \hat{N}_0 + I_x \right).
\]

A mobile is in outage with respect to a NodeB if the required transmit power exceeds the maximum transmit power. Thus, the probability of outage at location \( f \) for service \( s \) with respect to NodeB \( x \) is

\[
P_{\text{out}}(s,f,x) = P \left( \omega_s \cdot \hat{\theta} \cdot \left( W \cdot \hat{N}_0 + I_x \right) > \hat{S}_{f,s,x}^* \right).
\]

Note that \( \hat{\theta} \) is a lognormal random variable such that \( 1/\hat{\theta} \) follows the same distribution. Due to the soft handover a mobile is connected to all surrounding NodeBs such that it experiences outage only if it is in outage with respect to all NodeBs. Consequently, the total outage probability is

\[
\hat{P}_{\text{out}}(s,f) = \prod_{x \in B} P_{\text{out}}(s,f,x).
\]

Note that we neglect the correlations among the interferences at different NodeBs in this equation. As mentioned above we define the \( p \)-percent coverage area for a service as the set of all area elements with an outage probability of less than \( p \)-percent. The \( p \)-percent coverage area of service \( s \) is

\[
\mathcal{F}_{\text{cov}}(s,p) = \{ f | P_{\text{out}}(s,f) <= p \}.
\]

The outage probability at a certain location with respect to a NodeB \( x \) depends on three random variables: the service load \( \omega_s \), the shadow fading \( \hat{\theta} \), and the interference \( I_x \). The probability density function (PDF) of the service load is given by transforming the PDF of the received \( E_b/N_0 \) value \( \epsilon_s \) as in [9]. The shadow fading is defined to be lognormal. This leaves the computation of the PDF of the interference \( I_x \) which is the focus of the next section. The approach to assume that the interference is lognormally distributed such that we have to compute only the mean and the variance. If the PDFs of all random variables are known, Eq. (4) is transformed to the logarithmic scale and the outage probability follows by convolving the transformed PDFs.

III. INTERFERENCE CALCULATION

In this section we describe the basic idea of the algorithm for computing the mean and the variance of the interferences at the NodeBs in a WCDMA network without going into the details. A description at full length is given in [7]. The starting point of our analysis is the work in [8] where the basic relationship [8] between the interferences, other-cell interferences, and cell loads is derived from the basic power control equations.
For formulating this relationship we introduce some more variables. The key parameter that defines the interaction of loads in a WCDMA network is the ratio of a mobile’s propagation loss to its own NodeB to the propagation loss of another NodeB where it produces other-cell interference. We define the propagation ratio of a mobile $k$ with respect to a NodeB $y$ as
\[
\Delta_{k,y} = \frac{d_{k,y}}{d_{k,B(k)}}.
\]
(7)

Note that this propagation ratio is always less or equal to one as otherwise the power controlling NodeB $x$ would change. We further introduce the random variable $\Delta_{x,y}$ which is a random variable for the propagation gain for an arbitrary mobile that is power-controlled by NodeB $x$ but without a specific location. Using the definition of the propagation ratio we define some more load variables:
\[
\omega_{k,y} = \omega_k \cdot \Delta_{k,y},
\]
(8)
\[
\eta_{x,y} = \sum_{k \in \mathcal{M}_x} \omega_{k,y},
\]
(9)
\[
\zeta_{x,y} = \frac{\eta_{x,y}}{\eta_{x,x}}.
\]
(10)

The interpretation of these variables is the following: The variable $\omega_{k,y}$ stands for the load that a mobile power-controlled by $x$ causes at another NodeB $y$. The variable $\eta_{x,y}$ is the load that all mobiles power-controlled by NodeB $x$ together cause at NodeB $y$. The load $\eta_{x,x}$ is equal to the own-cell load $\eta_{x}$ and corresponds the common uplink load definition in single cell models. The sense of the variables $\zeta_{x,y}$ is hardest to explain as there is no intuitive meaning. They somehow define the impact of the load of NodeB $x$ on the interference at another NodeB $y$. Their definition allows to formulate the relationship between interferences, other-cell interferences, and cell loads very compactly
\[
\hat{I}_x = \zeta_{x,x} \left( W \cdot \hat{N}_0 + \hat{I}^{oth}_x \right),
\]
(11)
\[
\hat{I}^{oth}_x = \sum_{y \neq x} \zeta_{y,x} \cdot \left( W \cdot \hat{N}_0 + \hat{I}^{oth}_y \right).
\]
(12)

The objective of our analysis is to derive the distribution of the interference of every NodeB as this gives us the probability that service $s$ is available at area element $f$. We approximate the interference by a lognormal random variable which reduces the problem to deriving its mean and variance. In this paper we describe only the algorithm for computing the mean other-cell interferences. The computation of the variances follows the same principle and the derivation is given in [7].

We obtain the mean other-cell interferences according to Eq. (11) as
\[
E\left[\hat{I}^{oth}_x\right] = \sum_{y \neq x} \left( W \cdot \hat{N}_0 + E\left[\hat{I}^{oth}_y\right]\right) E[\zeta_{x,y}].
\]
(13)

We compute the mean of the other-cell interference by combining these equations to a matrix equation. Let us therefore define the row vector
\[
E\left[\hat{I}^{oth}_x\right] = E\left[\hat{I}^{oth}_x\right] k
\]
(14)

and the matrix
\[
E\left[\zeta\right] = \begin{cases} E\left[\zeta_{x,y}\right] & , \text{if } x \neq y \\ 0 & , \text{if } x = y \end{cases}.
\]
(15)

Further, we define the row vector $\hat{N}_0$ by $\hat{N}_0[x] = W \cdot \hat{N}_0$. Then we can write Eq. (13) as
\[
E\left[\hat{I}^{oth}_x\right] = \hat{N}_0 E\left[\zeta\right] \left( I - E\left[\zeta\right]\right)^{-1}.
\]
(17)

The matrix $I$ is the identity matrix. From the mean other-cell interferences we obtain the mean interferences according to Eq. (11)
\[
E\left[I_x\right] = E\left[\zeta_{x,x}\right] \left( W \cdot \hat{N}_0 + E\left[\hat{I}^{oth}_x\right]\right).
\]
(18)

The part missing in the computation is the calculation of the mean of the random variables $\zeta_{x,y}$ which depend on the number of user power-controlled by NodeB $x$, their service, and their propagation ratio with respect to $y$. We reduce the computations of the expectation of $\zeta_{x,y}$ to the computation of the expectation of $\zeta_{x,x}$ and the expectation of the propagation ratios $\Delta_{x,y}$.

\[
E\left[\zeta_{x,y}\right] = E\left[\zeta_{x,x}\right] E\left[\Delta_{x,y}\right].
\]
(19)

In doing so we separate the spatial component from the traffic component.

A. Traffic Component

The traffic component $\zeta_{x,x}$ depends on the number of users per service at NodeB $x$. The mean number of users per area element $f$ is given and the probability that a mobile on area element $f$ is power-controlled by $x$ is
\[
p_{f,x} = P\left\{ d_{f,x} = \max_{y} (d_{f,y}) \right\}.
\]
(20)

Thus, we obtain the mean number of users with service $s$ power-controlled by $x$ as
\[
a_{x,s} = \sum_{f} p_{s} \cdot a_{f} \cdot p_{f,x}.
\]
(21)

As the sum of Poisson distributed random variables remains Poisson we obtain a Poisson distributed number of users of service $s$ at NodeB $x$ with mean $a_{x,s}$. Now, there exist several ways to determine the mean of $\zeta_{x,x}$. The fundamental of all
three ways is to calculate the PDF $\phi_{\eta_{x,x}}(t)$ of $\eta_{x,x}$ which directly yields
\[ E[\zeta_{x,x}] = \int_0^{1-\delta} \phi_{\eta_{x,x}}(t) \frac{1}{1-t} dt. \tag{22} \]
Note that the own-cell load $\eta_{x,x}$ has to be strictly smaller than one as otherwise the system capacity is exceeded. In the following we will write $\eta$ instead of $\eta_{x,x}$. The first method is to compute the PDF of $\eta$ through numerically convolving the PDFs of the service loads $\omega_s$ as done in [6]. The second and fastest way is to assume that $\eta$ follows a lognormal distribution and determine only its mean and variance as in [5]
\[ E[\eta] = \sum_{s=1}^{S} a_s \cdot E[\omega_s], \quad \text{VAR}[\eta] = \sum_{s=1}^{S} a_s \cdot \text{VAR}[\omega_s]. \tag{23} \]
A third method is originally based on the Kaufman-Roberts recursion [1], [4] used for computing state and blocking probability in multi-service networks. The methods reduce an $S$-dimensional state space to a one-dimensional state space where $S$ is the number of services with heterogeneous resource requirement that share a common resource. A state in the one-dimensional state space aggregates all $S$-dimensional states with equal resource occupation. The reduction to a one-dimensional state space leads to a considerably smaller computational effort for calculating state and blocking probabilities. In WCDMA systems we also have a multi-service network. The resource requirements correspond to the service loads $\omega$ and the commonly occupied resource is the own-cell load which has to stay strictly below 1. The difference to the multi-service networks is that with imperfections of power control we have to deal with stochastic resource requirements as the $\omega_s$ are random variables such that a one-dimensional state aggregates $S$-dimensional states with similar load distribution. For the exact algorithm, see [7].

### B. Spatial Component

The impact of the spatial distribution of the users and the propagation model is considered by the propagation ratios $\Delta_{x,y}$. If the required received power is assumed to be equal for all NodeBs in the network the propagation ratio as defined in Eq. (7) already corresponds to the other-cell interference in [10], [11]. Therefore, the algorithm for computing the expectation of the propagation ratio is similar to the computation of the other-cell interference in [11] except that we consider a discretized area with heterogenous spatial traffic distribution. The expectation of the propagation ratio for a mobile that is located at area element $f$ and power-controlled by NodeB $x$, is
\[ E[\Delta_{f,x,y}] = E \left[ \frac{d_{f,y}}{d_{f,x}} \right] = \max_j B(\hat{d}_{f,z}) \] \tag{24}
and the propagation ratio for a mobile with random location but power-controlled by $x$ is
\[ E[\Delta_{x,y}] = \sum_f \frac{\alpha_f}{\alpha_x} \cdot E[\Delta_{f,x,y}] \tag{25} \]

### IV. Numerical Results

We study the impact of the network load and the shadow fading constant on the 95% coverage area of an example network. The 95% coverage area of service $s$ is defined as the set of all area elements $f$ with $P_{\text{cov}}(s,f) < 0.95$. The NodeB locations and the spatial traffic distribution of the example network are shown in Fig. 1. Darker colors indicate a higher traffic density and brighter colors a lower traffic density. The service mix consists of 70% Service 1 ($R_1=12.2$kbps, $\xi_1 \sim N(5.5,1.2)$), 20% Service 2 ($R_2=64$kbps, $\xi_2 \sim N(4,1.2)$), and 10% Service 3 ($R_3=144$kbps, $\xi_3 \sim N(3,1.2)$). This leads to roughly equal offered loads per service. We scale the traffic intensity according the maximum loaded cell and define the offered system load as $\eta_{\text{max}} = \max_{x \in B} E \left[ \sum_{y \in B} \eta_{y} \right]$. The path gain follows according to the proposal in the 3GPP standard $\gamma_{f,x} = -128.1 - 37.6 \log_{10}\text{dist}(f,x)$ and the shadow fading constant in equal for all links $\theta_{f,x} = \theta$.

Figures Fig. 2-Fig. 5 show the 95% coverage area for offered system loads of 0.5 and 0.7 with shadow fading constants of 0dB and 6dB. The different colors indicate the coverage areas of the different services. Black means that no service is available, dark gray means that only Service 1 is available, in the medium gray areas Services 1 and 2 are available, and all three services are available in the bright gray colored area. We start with the scenario with 0.5 load and no shadowing in Fig. 2. The network provides coverage for all three services in the largest part of the area. Only at the edges and in two larger spots Service 2 and 3 are available with less than 5%. Service 1 is covered everywhere. Of special interest are the two spots and how they develop if the load and the shadow fading increase. Fig. 3 shows the coverage area with an increased load and Fig. 4 with stronger shadow fading. The load increase effects the color of the two outage spots, their size remains roughly the same. The area with coverage for all services shrinks only slightly but locations without coverage emerge. In contrast, the main impact of the stronger shadow fading is on the size of the outage spots and not on their color. The area with complete coverage shrinks visibly but at least Service 1 is available all over the area. Fig. 5 finally shows the coverage area with increased load and shadow fading. The area with coverage for all services is similar to Fig. 4 but
the colors are darker which means less availability for the higher data rate services. Compared to Fig. 3 the availability of Service 3 decreases considerably. On the other hand, the black spot in the middle of one outage area disappears which means that the coverage area of Service 1 with small data rates increases.

Thus, we can conclude that the load mainly affects the coverage area of the low data rate services while the shadow fading affects all services equally and even reduces the outage areas for small data rate services.

V. C ONCLUSION

In this paper we presented an efficient algorithm for approximating the coverage area for the uplink of a UMTS network. This is one of the main tasks in the radio network planning process. The crux of our algorithm is, first, that the other-cell interference is approximated through a lognormal distribution and, second, that we are able to separate the traffic component from the spatial component. The latter feature is beneficial in two ways. First, the traffic component can be pre-computed for a set of offered loads and service mixes and then be used in different areas. Second, the same area can be planned for different loads without re-calculating the spacial component. For an example network with heterogenous spatial traffic we studied the impact of system load and shadow fading on the coverage area. We could conclude that the system load mainly affects the coverage area of the low data rate services while the shadow fading affects all services equally. Due to soft handover, more shadow fading may even enlarge the coverage area of low data rate services.

REFERENCES